SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release Distribution Unlimited
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 95 0126
6a. NAME OF PERFORMING ORGANIZATION University of Missouri St. Louis (6b. OFFICE SYMBOL (If applicable))	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State, and ZIP Code) Dept. of Physics and Astronomy 8001 Natural Bridge Rd St. Louis, MO 63121	7b. ADDRESS (City, State, and ZIP Code) Bldg. 410 Bolling AFB Washington, DC 20332
8a. NAME OF FUNDING / SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NE	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER Grant Nr. AFOSR 89-0416
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2305; TASK NO. B5; WORK UNIT ACCESSION NO.

DTIC ELECTED MAR 28 1995 G

11. TITLE (Include Security Classification)

Quantum 1/f Noise in High Technology Applications Including Ultrasmall Structures and Devices

12. PERSONAL AUTHOR(S)

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13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final Technical Report	FROM 6/15/89 TO 2/14/94	May 14, 1994	

16. SUPPLEMENTARY NOTATION

17 COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

Quantum 1/f Noise Theory & Appl., 1/f Noise, Electr. Noise in Semiconductor Devices, Quantum 1/f Effect, Quartz Resonators, Noise in Ultrasmall Dev., Chaos, Infrared Detectors, Nonlin. Dynamics

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The present report brings a final answer to the question on the nature of fundamental 1/f noise and its ubiquity. A sufficient criterion for a 1/f spectrum in arbitrary chaotic nonlinear systems is derived for the first time. This criterion guarantees a 1/f spectrum for nonlinear systems which also satisfy a condition of mathematical homogeneity. Briefly stated, nonlinearity + homogeneity = 1/f noise. The criterion results because the 1/f spectrum reproduces itself in a self-convolution. Among the five examples to which the criterion is applied is also quantum electrodynamics (QED), resulting in quantum 1/f noise as a fundamental form of quantum chaos. Nonlinearity of the system of a charged particle and its field, plus the basic homogeneity of physical equations causes the criterion to predict the quantum 1/f effect. The simple universal quantum 1/f formula is applied to infrared detectors and yields quantum 1/f noise in the dark current, but not in the photo-generated current. The fractal dimension of quantum 1/f noise is determined on the basis of its quantum chaos definition and is obtained theoretically as a function of bandwidth in a simple model by applying the Grassberger-Procaccia-Takens algorithm to the quantum 1/f theory. The quantum 1/f effect is successfully applied to quartz resonators and bipolar junction transistors. Finally, the quantum 1/f mobility fluctuations are calculated in silicon and the coherent quantum 1/f effect is derived for the first time from a new QED propagator with branch-point singularity. This opens the way to better bridging the gap between coherent and conventional quantum 1/f noise in small and ultrasmall devices.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Gerald Witt	22b. TELEPHONE (Include Area Code) 202/767-4932
	22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

QUANTUM 1/f NOISE IN HIGH TECHNOLOGY APPLICATIONS INCLUDING ULTRASMALL STRUCTURES AND DEVICES

FINAL TECHNICAL REPORT

July 15, 1989 - December 14, 1993

AFOSR Grant #89-0416

May 14, 1994

Abstract

The fundamental fluctuations of physical cross sections and process rates in Quantum Electrodynamics are known as quantum 1/f effect, are limiting most high-technology applications, and are described by the simple quantum 1/f formula for the fractional spectral density, $S_{\delta\sigma/\sigma}(f)=2\alpha A/fN$. In spite of their basic character, they are a special case of quantum 1/f chaos in nonlinear systems, with $\alpha=e^2/\hbar c$, $A=(2/3\pi)(\Delta v/c)^2$, Δv the velocity change in scattering, and N the number of carriers used to define the scattering cross section σ. The present report provides for the first time a sufficient criterion for the presence of a 1/f spectrum in an arbitrary nonlinear system, proves its sufficient character, and exemplifies it for the current carriers in semiconductors, for electrons in metals, for cars on the highway, and for the nonlinear system of particles and field in electrodynamics. The report gives first principles analytical calculation results for the quantum 1/f mobility fluctuations in electronic materials such as Si and GaAs, derived on the basis of the quantum 1/f cross correlation formulas obtained in the previous AFOSR Grant 85-0130. These quantum 1/f results give the conventional quantum 1/f noise as an analytical function of temperature, applied field, and all the physical parameters of the semiconductor material, allowing for optimization of the material for any given application. The quantum 1/f cross-correlations derived by the author have been used to recalculate and to graph the quantum 1/f mobility fluctuations in Si and GaAs samples as a function of temperature and doping, in good agreement with the measurements of Tacano in Japan and Hooge in the Netherlands. The fractal dimension of band-limited quantum 1/f noise was determined for the first time and found to coincide with the number of octaves considered, in agreement with known experiments. This indicates a quantum chaos nature of 1/f noise in infrared detectors, and we expect this to be valid in most cases in semiconductors. The quantum 1/f theory was applied to a quartz resonator directly for the first time, providing both for an explanation of the observed 1/f frequency fluctuations and for optimization means. The practical application of the quantum 1/f effect to quartz resonators in this report allows us to understand and to extend the stability limits of quartz resonators. A fundamental breakthrough was performed through the first direct derivation of the coherent quantum 1/f effect from a special quantum-electrodynamic propagator, and from the author's general sufficient 1/f chaos criterion presented in this report. Finally, a suggestion is presented, which provides an explanation of the discrepancies which have been noticed in the collector current 1/f noise of bipolar junction transistors. Our new formula for collector 1/f noise in ultrasmall BJT's was found to agree reasonably with the experiment.

CONTENTS

I. INTRODUCTION

Progress has been achieved through this grant in the study of nonlinear systems which generate chaotic 1/f fluctuations, in the application of the Quantum 1/f Theory [1-13] to various materials used in small and ultrasmall electronic devices, in the application of the Quantum 1/f Theory to electronic devices and in the fundamental Quantum 1/f Theory itself.

The close similarity of the classical and quantum 1/f theories and the initial development of the quantum 1/f theory by the author out of his efforts to quantize his classical turbulence theory, have led to sustained efforts of the author aimed to integrate all his theories as various forms or realizations of a fundamental notion of chaos in nonlinear systems. During this grant period, these efforts finally beared fruits. A general sufficient criterion was formulated, allowing to identify the nonlinear systems which exhibit 1/f spectra. This criterion is presented in Sec. II below. It displays for the first time the general mathematical principle causing the ubiquitous 1/f spectrum in nature, science and technology. It is followed in Sec. III by examples, in which the new criterion is applied to the classical and quantum mechanical forms of the author's 1/f noise theory. These five examples clarify the physical meaning of the new criterion.

Infrared detectors have provided a testing ground for the quantum 1/f theory in the eighties, and have been the subject of several quantum 1/f studies. In Sec. IV we present a practical application of the quantum 1/f theory to infrared detectors in the presence of radiation, in particular, a proof of the absence of quantum 1/f noise in the process of carrier photogeneration in semiconductors. Quantum 1/f noise is nevertheless predicted by the quantum 1/f theory and is present both in junction and MIS photodetectors, in the dark current and in the resistance of the homogeneous part of the semiconductors which form the detector.

Quantum 1/f noise was introduced as the quantum form of the classical turbulence process which is a form of chaos. Quantum 1/f noise is therefore a fundamental and universal form of quantum chaos. The first calculation of the fractal dimension of the quantum 1/f noise process presented in Sec. V.

The quantum 1/f theory has inspired the explanation of the origin of the famous Q^{-4} law of flicker of frequency in quartz crystal resonators by the author, as early as 1978. He has now developed a new method allowing for direct application of the quantum 1/f principle to quartz resonators and to other piezoelectric and ferroelectric systems, which is presented in Sec. VI. That section includes the successful practical application of the quantum 1/f theory to quartz resonators and the verification of the results at NIST-Boulder with the cooperation of Fred Walls. The quantum 1/f effect appears in the process rate of phonon removal from the main oscillator mode of the quartz crystal, because there is bremsstrahlung associated with this

process in any piezoelectric crystal. Indeed, as we shall see in Sec. VI, the process will suddenly change not only the velocity of the lattice vibration, but also the rate of chance of the polarization of the crystal, which in turn is equivalent to sudden electric current change.

We have tried to improve the application of quantum 1/f theory to the collector noise of bipolar transistors. This short calculation is presented in Sec. VII which also provides a comparison of our improvements in the quantum 1/f collector noise formula in ultrasmall BJTs with the experimental evidence.

For the practical application of the Quantum 1/f Theory it is necessary to derive the quantum 1/f fluctuations of various kinetic (transport) coefficients which characterize the materials used in electronic and microelectronic applications (mobility, diffusion coefficient), from the author's fundamental quantum 1/f formula. The latter is applicable only to cross sections and rates of elementary processes. Most important is the calculation of mobility fluctuations in Si, GaAs and $Hg_{1-x}Cd_xTe$. An earlier calculation (Kousik, Van Vliet, Handel, 1985) of mobility fluctuations in Si and GaAs is replaced in Sec. VIII by a more rigorous calculation, based on the new quantum 1/f cross-correlations, developed under the previous AFOSR Grant, and presented in the Final Technical Report AFOSR -85-0130. The new calculation yields increased 1/f noise, and is in vey good agreement with the experiment. The results give the quantum 1/f noise present in various carrier scattering mechanisms as a function of temperature and doping level.

The progress achieved in the fundamental quantum 1/f theory includes the direct derivation of the Coherent Quantum 1/f Effect from a special quantum-electrodynamical propagator known since 1975, and from the sufficient 1/f criterion introduced in the 1992 (Third) Annual Report (Sec. II). The physical meaning of the coherent quantum 1/f effect is displayed by this new independent derivation as a consequence of the indefinite energy of a physical electron.

At the same time, many new contributions to quantum 1/f theory and experiment were published by other workers in the field, considerably advancing the field of infra-quantum physics and quantum 1/f noise in high-technology applications. These new contributions, as well as new PhD thesis work in this field and contributions presented at the "4th Conference on Quantum 1/f Noise and other Low-Frequency Fluctuations" (MInneapolis 1990), at the 5th (St. Louis, 1992) and at the X, XI and XII Int. Conf. on Noise in Physical Systems (Budapest, 1989, Kyoto, 1991, St. Louis, 1993) are all included in the updated General Quantum 1/f Bibliography appended to this Report. My collaborators have been A.L. Först-Chung, E. Bernardi, T. Chung, L.M.N. Sastri, X. Hu, Jian Xu, Y. Zhang, and M. Leong.

II. SUFFICIENT CRITERION FOR 1/F NOISE IN CHAOTIC NONLINEAR SYSTEMS

Consider a n-dimensional nonlinear system described in terms of the dimensionless function $\mathbf{Y}(\mathbf{x},t)$ by the m^{th} order nonlinear dynamical equation

$$d\mathbf{Y}/dt + F(\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/dx_1 ... d\mathbf{Y}/dx_n, d^2\mathbf{Y}/dx_1{}^2 ..... d^m\mathbf{Y}/dx_n{}^m) = 0 \qquad (1)$$

If

$$F[\lambda\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/(\lambda dx_1) ... d\mathbf{Y}/(\lambda dx_n), d^2\mathbf{Y}/(\lambda dx_1)^2 ..... d^m\mathbf{Y}/(\lambda dx_n)^m]$$
$$= \lambda^p F(\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/dx_1 ... d\mathbf{Y}/dx_n, d^2\mathbf{Y}/dx_1{}^2 ..... d^m\mathbf{Y}/dx_n{}^m) \qquad (2)$$

for any real number λ, Eq. (1) is said to be homogeneous. Performing a Fourier transformation with respect to the vector $\mathbf{x}$, we get in terms of the Fourier-transformed wavevector $\mathbf{k}$ the nonlinear integro-differential equation

$$d\mathbf{y}(\mathbf{k},t)/dt + G[\mathbf{k}, \mathbf{y}(\mathbf{k},t), k_1\mathbf{y}(\mathbf{k},t) ... k_n\mathbf{y}(\mathbf{k},t), k_1{}^2\mathbf{y}(\mathbf{k},t) ..... k_n{}^m\mathbf{y}(\mathbf{k},t)] = 0, \qquad (3)$$

where $\mathbf{y}(\mathbf{k},t)$ is the Fourier transform of $\mathbf{Y}(\mathbf{x},t)$. Due to Eq. (2), G satisfies the relation

$$G[\lambda\mathbf{k}, \mathbf{y}, \lambda k_1\mathbf{y} ... \lambda k_n\mathbf{y}, (\lambda k_1)^2\mathbf{y} ..... (\lambda k_n)^m\mathbf{y}]$$
$$= \lambda^p G[\mathbf{k}, \mathbf{y}, k_1\mathbf{y} ... k_n\mathbf{y}, k_1{}^2\mathbf{y} ..... k_n{}^m\mathbf{y}]. \qquad (4)$$

Eq. (3) can thus be rewritten in the form

$$d\mathbf{y}/d(t/\lambda^p) + G[\lambda\mathbf{k}, \mathbf{y}, \lambda k_1\mathbf{y} ... \lambda k_n\mathbf{y}, (\lambda k_1)^2\mathbf{y} ..... (\lambda k_n)^m\mathbf{y}] = 0, \qquad (5)$$

Taking $\lambda=1/k$, where $k=|\mathbf{k}|=(x_1{}^2+....+x_n{}^2)^{1/2}$, and setting $k^pt=z$, we notice that k has been eliminated from the dynamical equation, and only $\mathbf{k}/k$ is left. This means that there is no privileged scale left for the system in $\mathbf{x}$ or $\mathbf{k}$ space, other than the scale defined by the given time t, and expressed by the dependence on z. We call this property of the dynamical system "*sliding-scale invariance*".

In certain conditions, instabilities of a solution of Eq. (1) may generate chaos, or turbulence. In a sufficiently large system described by the local dynamical equation (1), in which the boundary conditions become immaterial, homogeneous, isotropic turbulence, (chaos) can be obtained, with a spectral density determined only by Eq. (1). The stationary autocorrelation function $A(\tau)$ is defined as an average over the turbulent ensemble

$$A(\tau) = \langle \mathbf{Y}(\mathbf{x},t)\mathbf{Y}(\mathbf{x},t+\tau)\rangle = \int \langle \mathbf{y}(\mathbf{k},t)\mathbf{y}(\mathbf{k},t+\tau)\rangle d^n k = \int u(k,z) d^n k \qquad (6)$$

Here we have introduced the scalar

$$u(k,z) = \langle \mathbf{y}(\mathbf{k},t)\mathbf{y}(\mathbf{k},t+\tau)\rangle \qquad (7)$$

of homogeneous, isotropic chaos (turbulence), which depends only on $|\mathbf{k}|$ and $z=k^p\tau$. All integrals are from minus infinity to plus infinity. The chain of integro-differential equations for the correlation functions of any order obeys the same sliding-scale invariance which we have noticed in the fundamental dynamical equation above. *Therefore, in isotropic, homogeneous, conditions, u can only depend on k and z.* Furthermore, the direct dependence on k must reflect this sliding-scale invariance, and is therefore of the form

$$u(k,z) = k^{-n} v(z). \qquad (8)$$

Indeed, only this form insures that $u(k,z)d^n k$ and therefore also the corresponding integrals and multiple convolutions in k space have the necessary sliding-scale invariance.

According to the Wiener-Khintchine theorem, the spectral density is the Fourier-transform of A(t),

$$S_y(f) = \int e^{2\pi i f\tau} A(\tau) d\tau = (1/f) \int e^{2\pi i t'} \int k'^{-n} v(z) d^n k' dt' = C/f, \qquad (9)$$

where we have set $f\tau=t'$, $k^n=fk'^n$, $z=k^n\tau=k'^n t'$, and the integral

$$C = \int e^{2\pi i t'} \int k'^{-n} v(z) d^n k' dt' = \int e^{2\pi i t'} \int k''^{-n} v(k''^n) d^n k'' dt' \qquad (10)$$

is independent of f. We have defined the vector $k''=t'^{1/n}\,\mathbf{k}$.

In conclusion, we have shown that if the equation

$$dY/dt + F(\mathbf{x}, Y, dY/dx_1 ... dY/dx_n, d^2Y/dx_1^2 ..... d^mY/dx_n^m) = 0 \qquad (11)$$

with

$$F[\lambda\mathbf{x}, Y, dY/(\lambda dx_1) ... dY/(\lambda dx)_n, d^2Y/(\lambda dx_1)^2 ..... d^mY/(\lambda dx_n)^m] = \lambda^p F(\mathbf{x}, Y, dY/dx_1 ... dY/dx_n, d^2Y/dx_1^2 ..... d^mY/dx_n^m) \qquad (12)$$

admits, in the limit of weak dissipation, quasistationary homogeneous isotropic chaotic (turbulent) solutions which are practically independent of the nature of the instabilities or

bifurcations (or even stirring forces) which have caused the chaotic state, the corresponding spectral density must be proportional to 1/f.

We note that the solution (7) leads to a weak (logarithmic) divergence of the integral over k in the last form of Eq. (6) and in Eq. (9). This seems to contradict the fact that in practical applications the autocorrelation function A(t) is finite, and its value at t=0 is usually given in the problem at hand. However, in practice one never deals with an infinite volume, and the physical wave-vectors are also limited. For instance in fluid dynamics, wave vectors exceeding the reciprocal average distance between neighboring fluid molecules correspond to thermal motions, and are therefore no longer meaningful for the hydrodynamic treatment. Due to its logarithmic character, the divergnce is thus without practical importance. Nevertheless, for a given level of chaos A(0), we can construct an approximate solution

$$u(k,z)=k^{r-n}v(z), \tag{13}$$

with $0<r<<1$, which avoids the divergence at k=0. To get the correct chaos level with $k<k_0$, k_0 being an upper cutoff, we set

$$u=r[A(0)/v(0)]k_0^{-r}k^{r-n}v(z). \tag{14}$$

This yields for t=0 the result A(0) when we integrate over d^nk with an upper limit k_0. We notice that r is present both as a general factor, and as a small defect in the exponent of k. This is a general feature, present both in classical and quantum nonlinear systems with 1/f noise. In the limit $r\rightarrow 0$, the approximate solution tends to become exact. In the classical homogeneous, isotropic, turbulence theory, r can be arbitrarily small, while in the quantum 1/f theory (quantum electrodynamics), $r = \alpha A = (2\alpha/3\pi)(\Delta v/c)^2 << 1/137$ is the well-defined infrared exponent of the process, with $\alpha=e^2/\hbar c=1/137$, as we know from the theory of infrared radiative corrections. This allows us to formulate the justification of our criterion in a way used by the author for the first time in 1981 in an unpublished paper which was reviewed in that year by T. Musha and C.M. Van Vliet: Homogeneous nonlinear systems require the 1/f spectrum because this is the only one which is idempotent with respect to the self-convolution operation in the limit of very small r values. Indeed, the result of the multiple self-convolutions introduced by the Fourier transformation of a nonlinear term is again $1/f^{1-r}$ with a very small r, if the input was a $1/f^{1-r}$ with a very small r.

III. EXAMPLES ILLUSTRATING THE APPLICATION OF THE GENERAL SUFFICIENT CRITERION FOR 1/f NOISE IN CHAOTIC NONLINEAR SYSTEMS

In spite of the practical success of our quantum 1/f theory in explaining electronic 1/f noise in most high-tech devices, and in spite of the conceptual success of our earlier classical turbulence approach to 1/f noise, the question about the ultimate origin of nature's omnipresent 1/f spectra remained unanswered. During the last three decades, we have claimed repeatedly that nonlinearity is a general cause of 1/f noise. Our new criterion proves that nonlinearity always leads to a 1/f spectrum if homogeneity is also present in the equation(s) of motion. We can present this criterion, derived in the preceeding section, in a more general form. Specifically, let the system be described in terms of the dimensionless vector function $\mathbf{Y}(\mathbf{x},t)$ by the m^{th} order nonlinear system of differential equation

$$\Phi[t, \mathbf{x}, \mathbf{Y}, \partial\mathbf{Y}/\partial t, \partial\mathbf{Y}/\partial x_1 ... \partial\mathbf{Y}/\partial x_n, \partial^2\mathbf{Y}/\partial t^2, \partial^2\mathbf{Y}/\partial x_1{}^2 ..... \partial^m\mathbf{Y}/\partial x_n{}^m] = 0 \qquad (1')$$

where the vector function Φ may be nonlinear in any of its arguments. If a number θ exists such that Eq. (1') implies

$$\Phi[\lambda^\theta t, \lambda\mathbf{x}, \mathbf{Y}, \partial\mathbf{Y}/\lambda^\theta\partial t, \partial\mathbf{Y}/\lambda\partial x_1 ... \partial\mathbf{Y}/\lambda\partial x_n, \partial^2\mathbf{Y}/\lambda^{2\theta}\partial t^2, \partial^2\mathbf{Y}/\lambda^2\partial x_1{}^2 ..... \partial^m\mathbf{Y}/\lambda^m\partial x_n{}^m) = 0 \qquad (2')$$

for any real number λ, the power spectral density of any chaotic solution for the vector function $\mathbf{Y}$ defined by Eq. (1') is proportional to 1/f.

In conclusion, nonlinearity + homogeneity = 1/f noise. The ultimate cause of the ubiquitous 1/f noise in nature is the omnipresence of nonlinearities (no matter how weak) and homogeneity. The latter is finally related to rotational invariance and to the isotropy of space. All our four specific theories of 1/f chaos in nonlinear systems are just special cases to which our criterion is applicable. They include our magneto-plasma theory of turbulence in intrinsic symmetric semiconductors (1966, [14-16]), our similar theory for metals (1971, [17]), the quantum 1/f theory (pure quantum electrodynamics, 1975, [1-13]), and the theory of Musha's traffic turbulence (1989). A fifth, negative, application example concerns a one-dimensional crystal, i.e., a chain of atoms with slightly anharmonic interaction potentials, in which the criterion indicates the absence of a lattice-dynamical 1/f spectrum, in agreement with computer simulations by Musha and with analytical calculations by Koch and Wolf.

The general criterion developed in the preceeding section will now be illustrated on the basis of the examples mentioned above.

III.1 Classical Turbulence Theory for the Current Carriers in Semiconductors

In the case of homogeneous, isotropic turbulence [14-17] caused in the electron-hole plasma of an infinite sample of a symmetric intrinsic semiconductor by dynamical instabilities of any kind, we start from the equations

$$\nu \mathbf{v}^{+} = (e/2c)\mathbf{v}^{-}\times\mathbf{B} - (1/n)\nabla P, \tag{15}$$

$$\nu \mathbf{v}^{-} = 2e[E + \mathbf{v}^{+}\times\mathbf{B}/c] - (2/n)\nabla(P_p - P_n), \tag{16}$$

$$\nabla\cdot\mathbf{v}^{+} = 0 \qquad (n = \text{const}), \tag{17}$$

$$\nabla\times E = -(1/c)\partial B/\partial t, \tag{18}$$

$$\nabla\times B = 2\pi e n\mathbf{v}^{-}/c, \tag{19}$$

$$\nabla\cdot B = 0. \tag{20}$$

Here n is the total carrier concentration including an equal number of electrons and holes, ν/e their reciprocal mobility assumed to be the same for electrons and holes, P_n and P_p the partial pressures of electrons and holes, P the total carrier pressure, $2\mathbf{v}^{+}$, $\mathbf{v}^{-}$ the sum and the difference of the carrier drift velocities. Inertial terms proportional to the effective masses of the carriers, as well as electrostatic terms and compressibility terms have been neglected here in a consistent way [14-17], because we are interested in the low-frequency domain only. Although we do not work this out here, this system of equations can be shown to admit an energy theorem. Performing a Fourier expansion, we obtain

$$\nu \mathbf{v}^{+}(\mathbf{k}) = (e/2c)\sum_{\mathbf{k}'}\mathbf{v}^{-}(\mathbf{k}')\times\mathbf{B}(\mathbf{k}-\mathbf{k}') - (i/n)\mathbf{k}\cdot P(\mathbf{k}), \tag{21}$$

$$\nu \mathbf{v}^{-}(\mathbf{k}) = 2e[E(\mathbf{k}) + \sum_{\mathbf{k}'}\mathbf{v}^{+}(\mathbf{k}')\times\mathbf{B}(\mathbf{k}-\mathbf{k}')/c] - (2i/n)\mathbf{k}(P_p - P_n), \tag{22}$$

$$\mathbf{k}\cdot\mathbf{v}^{+}(\mathbf{k}) = 0. \tag{23}$$

$$i\mathbf{k}\times E(\mathbf{k}) = -(1/c)\partial B(\mathbf{k})/\partial t, \tag{24}$$

$$i\mathbf{k}\times\mathbf{B}(\mathbf{k}) = 2\pi e n \mathbf{v}^{-}(\mathbf{k})/c, \tag{25}$$

$$\mathbf{k}\cdot\mathbf{B}(\mathbf{k}) = 0, \tag{26}$$

Substituting **E** from Eq. (22) into Eq. (24), we obtain with Eq. (25)

$$\partial \mathbf{B}(\mathbf{k})/\partial t + \mu k^2 \mathbf{B}(\mathbf{k}) = i\mathbf{k}\times \sum_{\mathbf{k}'} \mathbf{v}^{+}(\mathbf{k}') \times \mathbf{B}(\mathbf{k}-\mathbf{k}'), \tag{27}$$

where $\mu = c^2\nu/4\pi n e^2$. Eqs. (21) and (25) yield

$$\begin{aligned}\mathbf{v}^{+}(\mathbf{k}) = (i/4\pi\nu n) \sum_{\mathbf{k}''} \Big\{ & \mathbf{B}(\mathbf{k}'')[\mathbf{k}''\cdot\mathbf{B}(\mathbf{k}-\mathbf{k}'')] - \mathbf{k}''[\mathbf{B}(\mathbf{k}'')\cdot\mathbf{B}(\mathbf{k}-\mathbf{k}'')] \\ & -(\mathbf{k}/k^2)(1-\delta_{\mathbf{k},0})[\mathbf{k}''\cdot\mathbf{B}(\mathbf{k}-\mathbf{k}'')\ \mathbf{k}\cdot\mathbf{B}(\mathbf{k}'') - \mathbf{B}(\mathbf{k}'')\cdot\mathbf{B}(\mathbf{k}-\mathbf{k}'')\ (\mathbf{k}\cdot\mathbf{k}'')]\Big\}\ .\end{aligned} \tag{28}$$

Substituting this into Eq. (27), we obtain the fundamental dynamical field-equation of turbulence in the electron-hole plasma of a symmetric intrinsic semiconductor

$$\begin{aligned}\partial b_\beta(\mathbf{k},t)/\partial t + \mu k^2 b_\beta(\mathbf{k},t) = & \sum_{\mathbf{k}'\mathbf{k}''} b_j(\mathbf{k}-\mathbf{k}',t)\ b_l(\mathbf{k}'',t)\ b_m(\mathbf{k}'-\mathbf{k}'',t) \\ & \cdot(k_j\delta_{\beta s}-k_s\delta_{\beta j})[k_s\delta_{lm}-k_m''\delta_{ls} + (k_s'/k'^2)(1-\delta_{\mathbf{k}',0})(k_m''k_l'-\mathbf{k}'\cdot\mathbf{k}''\delta_{lm})],\end{aligned} \tag{29}$$

in terms of $\mathbf{b} \equiv \mathbf{B}/\sqrt{2\pi\nu n}$. This dynamical equation has the form of Eq. (3), with p=2 in Eq. (4), and with G defined as the r.h.s. minus the term in k^2 on the l.h.s.. Our sufficient criterion thus tells us that this nonlinear system will yield a 1/f spectrum. We present below the direct derivation for this example.

Multiplying Eq. (29) with $b_\alpha^*(\mathbf{k},t-\tau)$ and taking the average over a statistical ensemble which represents our notion of stationary turbulence, we obtain in quasi-stationary conditions

$$\begin{aligned}&\partial w_{\alpha\beta}(\mathbf{k},\tau)/\partial\tau + \mu k^2 w_{\alpha\beta}(\mathbf{k},\tau) \\ &= \sum_{\mathbf{k}'\mathbf{k}''} \langle b_\alpha^*(\mathbf{k},t-\tau) b_j(\mathbf{k}-\mathbf{k}',t) b_l(\mathbf{k}'',t) b_m(\mathbf{k}'-\mathbf{k}'',t)\rangle R_{jml\beta},\end{aligned} \tag{30}$$

with $w_{\alpha\beta}(\mathbf{k},\tau) \equiv (L/2\pi)^3 \langle b_\alpha^*(\mathbf{k},t-\tau) b_\beta(\mathbf{k},t)\rangle$, where L is the edge of the cubic normalization box, and

$$R_{jm|\beta} \equiv (k_j\delta_{\beta s} - k_s\delta_{\beta j})[k_s\delta_{|m} - k_m{}''\delta_{|s} + (k_s'/k'^2)(1-\delta_{\mathbf{k}',0})(k_m{}''k_{|}' - \mathbf{k}'\cdot\mathbf{k}''\delta_{|m})]. \qquad (31)$$

Multiplying Eq. (29) with more magnetic field components and averaging, we obtain equations connecting the fourth-order correlation tensor to the sixth-order tensor, and so on [14-17]. To end this infinite chain of equations for the correlation functions, we make a quasinormality assumption which expresses the fourth-order moment appearing in Eq. (30) according to the scheme

$$\langle ABCD\rangle = \langle AB\rangle\langle CD\rangle + \langle AC\rangle\langle BD\rangle + \langle AD\rangle\langle BC\rangle, \qquad (32)$$

valid if the four field components would approximate a joint normal distribution. This approximation does not alter the homogeneity of the system, which ultimately causes the 1/f spectrum. This approximation yields the closed equation

$$\partial w_{\alpha\beta}(\mathbf{k},\tau)/\partial\tau + \mu k^2 w_{\alpha\beta}(\mathbf{k},\tau)$$

$$= (2\pi/L)^3 w_{\alpha j}(\mathbf{k},\tau)\sum_{\mathbf{k}'} w_{|m}(\mathbf{k}',0) R_{jm|\beta}(\mathbf{k},\mathbf{k}'). \qquad (33)$$

Isotropic turbulence requires $w_{\alpha\beta} = A_1(k)\delta_{\alpha\beta} + A_2(k)k_\alpha k_\beta$, with coefficients A_1 and A_2 related through Eq. (26), yielding

$$w_{\alpha\beta}(\mathbf{k},\tau) = (1/2)[\delta_{\alpha\beta} - k_\alpha k_\beta/k^2]u(k,\tau), \qquad (34)$$

where $u(k,\tau) = \sum_\alpha w_{\alpha\alpha}(\mathbf{k},\tau)$. Therefore, the scalar correlation function $u(k,\tau)$ satisfies the dynamical equation of homogeneous, isotropic, stationary turbulence

$$\frac{\partial v(k,x)}{\partial|x|} + v(k,x) = -\frac{1}{2}v(k,x)\int\frac{d^3k'}{k'^3}\frac{k^2+\mathbf{k}\cdot\mathbf{k}'}{(\mathbf{k}+\mathbf{k}')^2}\left[1-\left(\frac{\mathbf{k}\cdot\mathbf{k}'}{kk'}\right)^2\right]v(k',0), \qquad (35)$$

where $v(k,x)\equiv k^{-3}u(k,\tau)$, and $x = \mu\tau k^2$ is a dimensionless variable replacing τ. We convince ourselves that the integral is independent of k, provided v(k,x) does not depend on its first argument, by setting $\mathbf{k}'/k \equiv \kappa$. This yields a solution. However, with $v=e^{-m|x|}$ we get a logarithmic divergence at $\kappa=0$. We look for an exact solution of the form [14-17]

$$v(k,x) = hk^{\varepsilon}e^{-|x|m(k)}, \quad \text{or } u(k,x) = (h/k^{3-\varepsilon})\, e^{-|x|m(k)}, \tag{36}$$

where m(k) is very close to a constant, almost independent of k, and h is a constant proportional to the intensity of the turbulence, or the turbulence level. Substituting this into Eq. (35) and performing the integration, we obtain a finite result only for $0<\varepsilon<2$:

$$m(k) = 1 + h\, r(\varepsilon)k^{\varepsilon}, \text{ with } r(\varepsilon)=[2\pi^2 \cot an(\varepsilon\pi/2)]/[(1-\varepsilon^2)(3-\varepsilon^2)]. \tag{37}$$

We notice that m(k) is indeed practically constant when $0<\varepsilon<<1$ is very small, arbitrarily small. The value $\varepsilon=0$ leads to a logarithmic divergence, but we can set $\varepsilon=0$ for practical purposes.

The spectral density corresponding to Eq. (36) with $\varepsilon=0$ is

$$w_{\alpha\beta}(\omega) = (1/\pi)\int_0^{\infty}\cos\omega\tau d\tau \int w_{\alpha\beta}(k,\tau)d^3k$$

$$= \frac{4}{3}\delta_{\alpha\beta}\int_0^{\infty}k^2dk\int_0^{\infty}d\tau\, u(k,\tau)\cos\omega\tau = \frac{4}{3}h\delta_{\alpha\beta}\int_0^{\infty}\frac{mkdk}{\omega^2+m^2k^2} = \frac{\pi}{3}\frac{h}{\omega}\delta_{\alpha\beta}. \tag{38}$$

This is a 1/f spectrum. At the low frequency end we do not get a divergent spectral integral, because the more exact form of the spectrum with a finite small $\varepsilon<<1$ is [14-17]

$$\int_0^{\infty}\frac{mk^{1+\varepsilon}dk}{\omega^2+m^2k^4} = \frac{1}{\omega^{1-\varepsilon/2}m^{\varepsilon/2}}\int_0^{\infty}\frac{x^{1+\varepsilon}dx}{1+x^4}, \tag{39}$$

which is proportional to $f^{\varepsilon/2-1}$. It is interesting to note that for $\varepsilon<<1$ cotan $\varepsilon\pi/2 = 2/\varepsilon\pi$ in Eq. (37), and that the value of ε calculated from Eq. (37) is therefore proportional to h, or to the intensity of the turbulence. This feature of the classical theory [14-17] is expressed with fascinating clarity in the quantum form of the theory, where ε is replaced by $2\alpha A$ which also appears as a intensity factor multiplying the quantum 1/f noise.

The essential element which led to the 1/f spectrum in the classical turbulence theory is the *nonlinearity* of the equations of motion, caused by the reaction of the electric currents back on themselves via the generated electromagnetic field. The same feedback reaction, via the electromagnetic field, also caused the nonlinearity in the quantum 1/f theory, and in QED in

general, leading in the same way to an identical 1/f spectrum, this time with a physically more meaningful $\varepsilon=2\alpha A$. This *nonlinearity* induces the coupling between various scales of turbulence and leads to the dynamical equilibrium between eddies of all sizes, expressed by the 1/f spectrum. In the $\varepsilon=0$, or $\alpha A=0$, limit, this dynamical equilibrium assumes both for the quantum case and for homogeneous, isotropic, turbulence in the unbounded semiconductor sample the simplest form, characterized by scale-homogeneity, or scale invariance. Indeed, replacing for $\varepsilon=0$ in Eqs. (35) and (36) k and k' by λk and $\lambda k'$, while also replacing τ by τ/λ^2, (or ω by $\lambda^2\omega$), Eq. (35) is not affected, and λ drops out. We conclude that *in the weak turbulence limit ($\varepsilon=0$) we obtain perfect self-similarity of the turbulence process at all scales in space and time, classically and quantum-mechanically.* The implied scale invariance is caused by the absence of any characteristic length or time scale, or by the presence of a sliding scale. Indeed, the frequency scale μk^2 is a function of the size of the eddies, given by the wave number k which can have any value. The actual frequency and wave-number spectra are closely related fractals, but in the weak-turbulence limit they approach an exact 1/f and $1/k^3$ spectrum respectively. In fact, we are here understanding the nonlinear dynamics which shapes this fractal for the first time.

III.2 Turbulence Theory for Drude Electrons in Metals

Our classical turbulence theory can be extended to the case of metals or degenerate extrinsic semiconductors [17] in the Drude free electron gas model. The system of integro-differential equations is quite different,

$$\nu \mathbf{v}^{+} = -e\mathbf{E} - (e/c)\mathbf{v}\times\mathbf{B} - (1/n)\nabla P \qquad (40)$$

$$\nabla\times\mathbf{E} = -(1/c)\partial\mathbf{B}/\partial t \qquad (41)$$

$$\nabla\times\mathbf{B} = -(4\pi e n/c)\mathbf{v} \qquad (42)$$

$$\nabla\cdot\mathbf{B} = 0, \qquad (43)$$

and leads to a third-order nonlinearity [17] in the resulting closed equation of motion, or nonlinear field-equation, which replaces Eq. (28):

$$\partial\mathbf{B}(\mathbf{k},t)/\partial t + \nu k^2\mathbf{B}(\mathbf{k},t) = -(c/4\pi n e)\mathbf{k}\times\int d^3k'\mathbf{B}(\mathbf{k}-\mathbf{k}',t)\times[\mathbf{k}'\times\mathbf{B}(\mathbf{k}',t)]. \qquad (44)$$

This is again in the form of Eq. (3), with p=2 in Eq. (4). We thus expecta 1/f spectrum in this system as well. This time we only sketch the derivation. The corresponding infinite chain of equations for the correlation tensors now goes in steps of one. As was shown above, it went in steps of two for semiconductors. The third-order correlation can be eliminated between the first and second equations in the chain. The resulting dynamical equation [17] for homogeneous, isotropic, stationary turbulence, which replaces Eq. (35), with the same notations, using $\mathbf{e}_3$ as the unit vector of the third axis, is

$$-\frac{\partial^2 v(k,x)}{\partial x^2} + v(k,x)$$
$$= -\frac{1}{2}\int\frac{d^3\kappa}{\kappa^3}\frac{1+\kappa\cdot\mathbf{e}_3}{|\mathbf{e}_3+\kappa|^3}\left[1-\frac{(\kappa\cdot\mathbf{e}_3)^2}{\kappa^2}\right]v(\kappa k,\kappa^2 x)v(k|\mathbf{e}_3+\kappa|,x|\mathbf{e}_3+\kappa|^2). \qquad (45)$$

This also admits, in the $\varepsilon=0$ limit of weak turbulence, a solution v(k,x) which does not depend on the first argument, and $u(k,x) = k^{-3}e^{-xm(x)}$, this time with an x-dependent m. With the change of variables $t=\omega\tau$ and $k'=k/\sqrt{\omega}$ in the second (middle, involving u) form of Eq. (38), x remains invariant, and a factor $1/\omega$ will appear in front of the integrals which themselves will just yield a constant factor independent of ω, τ or k. We thus obtain again a universal 1/f spectrum. As is shown in detail elsewhere [17], this 1/f spectrum is expressed in the corresponding current and voltage fluctuations which can be observed in the semiconductor or metallic medium. We conclude that the 1/f spectrum is a general property of electrically conducting systems in interaction with the electromagnetic field, a property which is caused by the nonlinearity of the system of carriers and field in mutual interaction due to the absence of a characteristic scale in the nonlinear equation of motion, and which finds its clearest expression in the Quantum 1/f Effect.

III.3 Theory of Highway Traffic Fluctuations

Musha and Higuchi [18,19] discovered the 1/f spectrum of highway traffic fluctuations empirically in 1977, also developing a model based on a postulated linear dependence of the average traffic speed v on the linear concentration of cars n(x) on the road, $v=v_0(1-n/n_s)$. The model was recognized to be similar to Burger's model of turbulence, and was simulated numerically leading to a 1/f-like spectrum at low frequencies.

The present paper develops a statistical turbulence theory for the Musha model, showing how in the low wave number and low frequency limit a universal 1/f spectrum emerges. In this limit, this is a particular case of the author's sliding-scale-invariant class of nonlinear

systems, all characterized by the presence of a universal 1/f spectrum. The author's classical and quantum 1/f theory is another example in the same class.

Musha writes the traffic current J as a sum of drift and diffusion currents

$$J = nv - D\partial n/\partial x = nv_o - (v_o/n_s)n^2 - D\partial n/\partial x, \qquad (46)$$

where D is a "diffusion coefficient". The equation of continuity is [8,9]

$$0 = \partial n/\partial t + \partial J/\partial x \equiv (\partial/\partial t + v_o\partial/\partial x)n - 2(v_o/n_s)n\partial n/\partial x - D\partial^2 n/\partial x^2. \qquad (47)$$

This is Musha's fundamental equation of traffic dynamics, which also was written [18] in a system of reference defined by

$$x' = -x + v_o t, \qquad t' = t \qquad (48)$$

in the final form [18]

$$\partial n/\partial t' + an\partial n/\partial x' = D\partial^2 n/\partial x'^2, \qquad (49)$$

where $a \equiv 2v_o/n_s$.

We normalize the concentration to $n_s/2$, the coordinate along the road x to D/v_o, and the time to D/v_o^2, thereby obtaining the dimensionless form of the fundamental traffic-dynamical equation

$$\partial n(x,t)/\partial t + n(x,t)\partial n(x,t)/\partial x = \partial^2 n(x,t)/\partial x^2, \qquad (50)$$

where we did not bother to change the notation, and returned to the original unprimed notation. Expanding in a Fourier series over the interval L, we obtain

$$\partial n/\partial t + i\Sigma_{k'}k'n(k')n(k-k') = -k^2 n(k,t). \qquad (51)$$

Comparing Eq. (50) with Eqs. (1),(2), or Eq. (51) with Eqs. (3),(4), we notice that this time our criterion is not satisfied, due to the r.h.s. term. However, in the limit of small k that term becomes negligible, and thus, our criterion becomes applicable, and we should get a 1/f spectrum in the low frequency limit. This would correspond to p=1 in Eqs. (2) and (3). We derive this 1/f spectrum below.

Defining the autocorrelation function

$$A(\xi,\tau) \equiv \langle n^*(x,t)n(x+\xi,t+\tau)\rangle = \Sigma_k \langle n^*(k,t)n(k,t+\tau)\rangle e^{ik\xi} = \Sigma_k U(k,\tau)e^{ik\xi}, \quad (52)$$

as a turbulent ensemble average $\langle\ldots\rangle$, in quasistationary homogeneous conditions we obtain by multiplication of Eq. (51) with $n^*(k, t-\tau)$, after ensemble averaging,

$$\partial U/\partial\tau + k^2 U(k,\tau) + i\Sigma_{k'} k' \langle n^*(k, t-\tau)n(k',t)n(k-k',t)\rangle = 0. \quad (53)$$

Linking the complex function $U(k,\tau)$ to a third-order correlation function, this is the first equation of an infinite chain of equations connecting the N^{th} order to the order N+1 correlation function. Applying the operator $-\partial/\partial\tau + k^2$ to Eq. (53), and using the complex conjugate of Eq. (51) to define the action of this operator on the first factor inside the averaging brackets in Eq. (53), we obtain

$$-\partial^2 U/\partial\tau^2 + k^4 U = -i\Sigma_{k'} k' \langle [i\Sigma_{k''} k'' n^*(k'',t-\tau) n^*(k-k'',t-\tau)] n(k',t)n(k-k',t)\rangle$$
$$= \Sigma_{k',k''} k'k'' [U(k'',0)U(k',0)\delta_{k,0} + U(k',\tau)U(k-k',\tau)\delta_{k''k'} + U(k',\tau)U(k-k',\tau)\delta_{k'',k-k'}]. \quad (54)$$

The last form was obtained by approximating the fourth-order correlation function with its expression in terms of the second-order correlation applicable for Gaussian processes:

$$\langle ABCD\rangle = \langle AB\rangle\langle CD\rangle + \langle AC\rangle\langle BD\rangle + \langle AD\rangle\langle BC\rangle. \quad (55)$$

This approximation was used by Heisenberg [20] in his turbulence theory, and was successfully verified in its practical applicability by Uberoi [21] and Batchelor [22].

In the limit $L\to\infty$, setting $U(k,\tau)L/2\pi = u(k\tau)$, we write for $k\neq 0$ Eq. (54) in the form

$$-\partial^2 u/\partial\tau^2 + k^4 u = \int [k'^2 u(k',\tau)u(k-k',\tau) + k'(k-k')u(k',\tau)u(k-k',\tau)]dk'$$

or

$$-\partial^2 u(k,\tau)/\partial\tau^2 + k^4 u(k,\tau) = k\int k' u(k',\tau)u(k-k',\tau)dk'. \quad (56)$$

This is our fundamental dynamical equation of traffic turbulence. All unspecified limits on integrals are from $-\infty$ to ∞, as we mentioned earlier. From the symmetry $A(\xi,\tau) = A(-\xi,-\tau)$

$=A^*(\xi,\tau)$ we see that a physically acceptable solution of this equation must satisfy the conditions

$$u(k,\tau) = u(-k,-\tau) = u^*(-k,-\tau) = u^*(k,-\tau). \qquad (57)$$

A solution of the form

$$u(k,\tau) = V(k)e^{-imk\tau}, \text{ with } V(k) = V(-k) = V^*(k) \qquad (58)$$

where m is a real constant, substituted into Eq. (56), yields

$$k(m^2 + k^2)V(k) = \int k'V(k')V(k-k')dk'. \qquad (59)$$

In the low wave number region $k<<1$ (i.e., $k<<v_o/D$) we neglect the k^2 term which arises from diffusion, and get

$$m^2kV(k) = \int k'V(k')V(k-k')dk'. \qquad (60)$$

A solution $V(k) = C|k|^{\varepsilon-1}$, with arbitrarily small $\varepsilon>0$, yields the value of C, independent of k only in the limit of $\varepsilon\to 0$:

$$m^2/C = \int_{-\infty}^{\infty} k'|k'|^{\varepsilon-1}|1-k'/k|^{\varepsilon-1}dk'/k = |k|^{\varepsilon}\int_{-\infty}^{\infty}\kappa|\kappa|^{\varepsilon-1}|1-\kappa|^{\varepsilon-1}d\kappa$$

$$= |k|^{\varepsilon}\int_{0}^{2}|\kappa|^{\varepsilon}|1-\kappa|^{\varepsilon-1}d\kappa = 2/\varepsilon = l = \text{const}, \qquad (61)$$

where we used the substitution $k'/k=\kappa$. The divergence at $\varepsilon=0$ disappears when we return to a finite value of the maximal road length L, which corresponds to a minimal k (or k') value $k_o=2\pi/L$, and transforms the integrals back into sums.

Eq. (61) establishes a proportionality between the level of the turbulence,described by C, and the magnitude of the small parameter ε as in earlier turbulence calculations [14-17]. The same fundamental feature is more clearly expressed, without the pseudo-singularities present here, in the quantum 1/f theory [1-13], where ε becomes the infrared exponent αA known from quantum electrodynamics. Indeed, αA is present there, just as we see it come in

here, both as a factor in front of the final result, and as a small defect in the exponent of the frequency. We also mention that the apparent absolute determination of C by Eq. (61) is just an artifact which reminds us that we omitted a source term in Eqs. (47), (49-51), (53-54), (56), which comes in as a δ function of time, $h(k)\delta(t)$ on the r.h.s. of Eq. (53), only if we assume the excitation in Eq. (47) to depend on time like white noise. Once that source term is written explicitly, it will allow us to determine C as this was done earlier [14] for a different equation, and it does not affect our equations for $\tau \neq 0$.

We can rewrite Eq. (52) in the form

$$A(\xi,\tau) \equiv \langle n^*(x,t)n(x+\xi,t+\tau)\rangle = \Sigma_k U(k,\tau)e^{ik\xi} \equiv \int dk\ u(k,\tau)e^{ik\xi}$$
$$= C\int e^{ik(\xi-m\tau)}|k|^{\varepsilon-1}dk = |\xi-m\tau|^{-\varepsilon}(m^2/I)\int e^{i\kappa}|\kappa|^{\varepsilon-1}d\kappa \approx m^2|\xi-m\tau|^{-\varepsilon}. \qquad (62)$$

Here we have used the substitution $k|\xi-m\tau|=\kappa$, and we have taken the limit $\varepsilon\to 0$ in the integral and in I, noting that they exhibit the same divergence, and therefore can be simplified in the limit. The cancellation of the divergences in the expression of the autocorrelation function shows that this important function is finite even in the continuum limit. It is only the spectral distribution of turbulence which exhibits a singularity at low wave numbers and frequencies.

Since only the limit $\varepsilon=0$ satisfies the fundamental dynamical equation of traffic turbulence, Eq. (62) means that the autocorrelation function is a constant. It also indicates that the constant has to be interpreted as the limit of a slowly decreasing power law which depends only on $|\xi-\tau|$; therefore the spectrum in wave numbers is the same as the spectrum in frequencies. The Fourier transform with respect to τ is the spectral density

$$S_n'(\xi,\omega) = \int \langle n^*(x,t)n(x+\xi,t+\tau)\rangle e^{i\omega\tau}d\tau = C\iint (dk/|k|^{1-\varepsilon})e^{ik(\xi-m\tau)+i\omega\tau}d\tau. \qquad (63)$$

To leave the moving (primed) frame and return to the system of reference at rest, we replace ω by $\omega-kv_o$ in the last integrand, actually just by $\omega-k$ in our ($v_o=1$) normalization:

$$S_n(\xi,\omega) = \int \langle n^*(x,t)n(x+\xi,t+\tau)\rangle e^{i(\omega-kv_o)\tau}d\tau$$
$$= C\iint (dk/k^{1-\varepsilon})e^{ik(\xi-m\tau)+i(\omega-k)\tau}d\tau. \qquad (64)$$

The spectral density of concentration fluctuations in a given point is obtained by setting $\xi=0$. The integration with respect to τ yields a delta function $2\pi\delta[\omega-(m+1)k]$, and we finally obtain the spectrum

$$S_n(0,\omega) = C[(m+1)/\omega]^{1-\varepsilon} \qquad (65)$$

where again the constant will turn out to be finite when we coarse grain the integrals. As mentioned above, the constant will actually be determined by the excitation term omitted in Eq. (56).

According to Eq. (2) we can write $J(k,\omega) = (\omega/k)n(k,\omega)$ in the system at rest. Therefore, for J we include a factor $(\omega/k)^2$ into the integrand of Eq. (64), and we get again the spectrum

$$S_J(\xi,\omega) = \int \langle J^*(x,t)J(x+\xi,t+\tau)\rangle e^{i(\omega-kv_o)\tau}d\tau$$
$$= C\iint(\omega^2 dk/k^{3-\varepsilon})e^{ik(\xi-m\tau)+i(\omega-k)\tau}d\tau; \qquad (66)$$
$$S_J(0,\omega) = C(m+1)^2\ [(m+1)/\omega]^{1-\varepsilon} = \text{Const}/\omega. \qquad (67)$$

Due to the neglect of the k^2 term in Eq. (59), the universal 1/f spectrum will be limited towards high frequencies by $\omega<v_o^2/D$, and the 1/k-spectrum by $k<v_o/D$. This limitation occurs because only Eq. (60) satisfies the condition of being free of any characteristic scale, thus exhibiting our sliding scale invariance [23], while Eq. (59) does not have this property. We have thus constructed a statistical dynamic theory of traffic turbulence, proving analytically Musha's earlier result, without any pretension of mathematical rigor. Traffic turbulence arising from instabilities of the laminar traffic flow can be considered as a form of classical fluid-dynamical chaos.

III.4 Quantum 1/f Noise: Application of the Sufficient 1/f Criterion to QED Quantum 1/f Chaos

The nonlinearity causing the 1/f spectrum of turbulence in both semiconductors and metals is caused by the reaction of the field generated by charged particles and their currents back on themselves. The same nonlinearity is present in quantum electrodynamics (QED), where it causes the infrared divergence, the infrared radiative corrections for cross sections and process rates, and the quantum 1/f effect. We shall prove this on the basis of the sufficient criterion for 1/f spectral density in chaotic systems, derived in the previous annual report.

Consider a beam of charged particles propagating in a well-defined direction which we shall call the x direction, so that the one-dimensional Schrödinger equation describes the longitudinal fluctuations in the concentration of particles. Considering the non-relativistic case

which is encountered in most quantum 1/f noise applications, we write in second quantization the equation of motion for the Heisenberg field operators ψ of the in the form

$$i\hbar\partial\psi/\partial t = (1/2m)[-i\hbar\nabla - (e/c)\,A]^2\psi, \qquad (68)$$

With the non-relativistic form $J = -i\hbar\psi^*\nabla\psi/m$ + hermitic conjugate, and with

$$A(x,y,z,t) = (\hbar/2cmi)\int\frac{[\psi^*\nabla\psi-\psi\nabla\psi^*]}{|x-x'|}dx' \qquad (69)$$

we obtain

$$i\hbar\partial\psi/\partial t = (1/2m)\left[-i\hbar\nabla - (e\hbar/2c^2mi)\int\frac{[\psi^*\nabla\psi-\psi\nabla\psi^*]}{|x-x'|}dx'\right]^2\psi. \qquad (70)$$

At very low frequencies or wavenumbers the last term in rectangular brackets is dominant on the r.h.s., leading to

$$i\hbar\partial\psi/\partial t = (-1/2m)\left[(e\hbar/2c^2m)\int\frac{[\psi^*\nabla\psi-\psi\nabla\psi^*]}{|x-x'|}dx'\right]^2\psi. \qquad (71)$$

For x replaced by λx, and x' replaced by $\lambda x'$, we obtain

$$i\hbar\partial\psi/\partial t = (-1/2m)\left[(e\hbar/2c^2m)\int\frac{[\psi^*\nabla/\lambda\psi-\psi\nabla/\lambda\psi^*]}{\lambda|x-x'|}\lambda^3dx'\right]^2\psi = \lambda^2H\psi = \lambda^{-p}H\psi. \qquad (72)$$

This satisfies our homogeneity criterion with p=-2. Our sufficient criterion only requires homogeneity, with any value of the weight p, for the existence of a 1/f spectrum in chaos. Therefore, we expect a 1/f spectrum of quantum current-fluctuations, i.e., of cross sections and process rates in physics, as derived in detail in Sec. II above. This is in agreement with the well-known, and experimentally verified, results of the Quantum 1/f Theory.

In conclusion, we realize that, both in classical and quantum mechanical nonlinear systems, the limiting behavior at low wave numbers is usually expressed by homogeneous functional dependences, leading to fundamental 1/f spectra on the basis of our criterion. Therefore, we expect a 1/f spectrum of current fluctuations, i.e., of cross sections and process rates in physics. This is in agreement with the well-known, and experimentally verified, results of the conventional Quantum 1/f Theory.

In conclusion, we realize that, both in classical and quantum mechanical nonlinear systems, the limiting behaviour at low wave numbers is usually expressed by homogeneous

functional dependences, (as shown in Eq. 2), leading to fundamental 1/f spectra on the basis of our criterion.

III.5 ABSENCE OF 1/F FLUCTUATION SPECTRA FOR MODE ENERGY AND PHONON NUMBER IN A NONLINEAR CHAOTIC CHAIN OF ATOMS

Consider a chain of atoms in the x direction, with a lattice constant b and displacements q_i from the equilibrium position. The equations of motion

$$m d^2q_n/dt^2 = A[(q_{n+1}-q_n)-(q_n-q_{n-1})] + B[(q_{n+1}-q_n)^2-(q_n-q_{n-1})^2] + C[(q_{n+1}-q_n)^3-(q_n-q_{n-1})^3] \quad (73)$$

contain anharmonic terms as long as B and C are different from zero. With $\alpha_n=q_{n+1}-q_n$ and $\beta_n=q_n-q_{n-1}$, neglecting the second-order term, we obtain

$$m d^2q_n/dt^2 = [(q_{n+1}-q_n)-(q_n-q_{n-1})][A + B(q_{n+1}-q_{n-1}) + C(\alpha_n^2+\alpha_n\beta_n+\beta_n^2)]. \quad (74)$$

Going over from finite differences to a continuum description, we obtain a differential equation for q(x,t)

$$m\partial^2q/\partial t^2 = b^2\partial^2q/\partial x^2[A + bB\partial q/\partial x + 3Cb^2(\partial q/\partial x)^2]. \quad (75)$$

Performing a fourier transform with respect to x,

$$m\partial^2q_k/\partial t^2 = -Ab^2k^2q_k - ib^2B\int(k-k')k'^2q_{k'}q_{k-k'}dk' + 3Cb^4\int dk'\int dk''k'k''(k-k'-k'')^2q_{k'}q_{k''}q_{k-k'-k''}. \quad (76)$$

All integrals are from minus infinity to infinity. Substituting $q_k=u(k,t)\exp[ikbt(A/m)^{1/2}]$,

$$m\partial^2u/\partial t^2 + 2ikb(A/m)^{1/2}\partial u/\partial t = -ib^3B\int(k-k')k'^2u_{k'}u_{k-k'}dk' +3Cb^4\int dk'\int dk''k'k''(k-k'-k'')^2u_{k'}u_{k''}u_{k-k'-k''}. \quad (77)$$

our 1/f chaos criterion requires both nonlinearity and homogeneity, as well as the presence of chaos or of a quasichaotic state. The nonlinearity condition is satisfied unless B=0 and C=0,

while homogeneity requires the existence of two numbers p and θ such that replacing k by λk everywhere except in the integration differentials, and replacing t by $\lambda^{\theta} t$ leaves the equation multiplied by a general factor λ^{P}, i.e., formally invariant. In our case we note that homogeneity would be is satisfied with p=2 and θ=-1 only if we neglect both the second-order and third-order terms by setting B=C=0, which would violate the nonlinearity requirement. In conclusion, our criterion can not be satisfied.

Nevertheless, in three-dimensional piezoelectric crystals quantum 1/f fluctuations of the phonon number are predicted by the electrodynamic quantum 1/f noise theory as we show in detail in Sec. VI, and have been observed experimentally in the Brillouin scattering of light by T. Musha. These quantum 1/f fluctuations are predicted by the sufficient criterion as we have seen in Sec. III.4.

IV. PRACTICAL APPLICATION TO QUANTUM 1/f NOISE IN INFRARED DETECTORS

Quantum 1/f Noise is a fundamental aspect of quantum mechanics, representing universal fluctuations of physical process rates R and cross sections σ given by the fractional (or relative) spectral density $S(f) = 2\alpha A/fN$. Therefore it is present in the process rates generating the dark current observed in junction photodetectors, such as *diffusion* (scattering cross sections fluctuate) in diffusion-limited junctions, and *recombination* in the recombination-limited regime. One is therefore tempted to expect similar fluctuations in the *photogeneration* of electron-hole pairs. However, as we show below, the corresponding quantum 1/f coefficient is zero, precluding the existence of quantum 1/f fluctuations in the photo-generation rate. Here N is the number of carriers used to define or measure the process rate or cross section considered.

For an arbitrary process involving a total of n incoming and outgoing charged particles, the nonrelativistic quantum 1/f coefficient is given [24] by

$$2\alpha A = (4\alpha/3\pi c^2) \sum_{i;j=1}^{n} \eta_i \eta_j q_i q_j (\mathbf{v}_i - \mathbf{v}_j)^2, \qquad (78)$$

where the summation runs over the charges q_i and velocities $\mathbf{v}_i$ of all incoming (η_i=-1) and outgoing (η_i=1) particles (altogether n of them) in the process whose quantum 1/f noise we want to find, and α is Sommerfeld's fine-structure constant, $e^2/\hbar c$=1/137. In a photoelectric process a photon (q=0) is absorbed, and a pair of oppositely charged particles is generated (η=1) with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ which are either zero, or quickly decay to zero in a time negligible with respect to the reciprocal frequency at which we calculate the quantum 1/f noise.

Thus in our case there are no incoming charged particles, and n=0+2=2. The αA coefficient of a photogeneration process is therefore zero,

$$\alpha A_{ph} = (1,1)+(2,2)+(1,2)+(2,1) = 0+0+ (4\alpha/3\pi c^2)(\mathbf{v}_1-\mathbf{v}_2)^2 \approx 0. \qquad (79)$$

All photogenerated carriers of the right sign are collected in the well of the charge-coupled device, although they may generate quantum 1/f voltage fluctuations on their way. Since usually only the number of carriers collected at read-out matters, no quantum 1/f noise will be observed in a photoelectric CCD as long as the dark current is negligible with respect to the photocurrent. This is in agreement with the experiments performed by Mooney [25] of RADC-Hanscom AFB. The same considerations apply to Metal-Insulator-Semiconductor (MIS) photodetectors [26].

V. QUANTUM 1/F NOISE IS QUANTUM CHAOS: FRACTAL DIMENSION OF QUANTUM 1/F NOISE

Quantum 1/f noise in a physical quantity such as a current j, a cross section, a process rate, or a kinetic coefficient, such as the mobility, is represented by an expression of the form

$$\delta j/\langle j\rangle = a \sum_{\mathbf{k};\lambda} |b(\mathbf{k},\lambda)|^2 \cos(ckt+\gamma_{k\lambda}) \qquad (80)$$

where the sum is over all electromagnetic modes labeled by their wave vector $\mathbf{k}$ and polarization λ, with $k=|\mathbf{k}|$. Eq. (80) is deterministic and the particle-specific random initial phases $\gamma_{k\lambda}$ are present in each term as initial conditions, like the initial phases describing a turbulent fluid are present in each Fourier component of the perfectly deterministic velocity field. The difference between the mentioned classical chaos (turbulence) case and the quantum 1/f chaos we are introducing here becomes evident when we recall that J is in fact a probability current density! Can we calculate a spectral density of the fluctuations δj of this probability current and claim that it represents the expectation value of the spectral density of the observed quantum 1/f fluctuations? Our rigorous derivation in second quantization [23, 28-29] tells us we can, provided we divide the single-particle result by N in the case of bosons and N-1 for fermions. Here N is the number of scattered particles used to define what we call scattered current j or scattering cross section σ. Encouraged by this result, we have undertaken an effort to determine the fractal dimension of quantum 1/f noise given by Eq. (80) and restricted to an observed frequency interval from $ck_o= \varepsilon_o=2\pi f_o$ to $\Lambda=2\pi F$, where $\varepsilon_o=2\pi/T$ is determined by the duration T of the 1/f nose measurement. Our objective is to determine the fractal dimension of

quantum 1/f noise and to compare it with measurements of the fractal dimension of physical 1/f noise in HgCdTe MWIR photodiodes [30]. Calculating the spectral density, replacing the summation in Eq. (80) by an integral, and going back to the original fluctuations, we obtain an equivalent representation of the fluctuations [23,28-29]

$$\delta j/<j> = 2(\alpha A)^{1/2} \int_{\varepsilon_0}^{\Lambda} \cos(\varepsilon t+\gamma_\varepsilon) d\varepsilon/\sqrt{\varepsilon} \qquad (81)$$

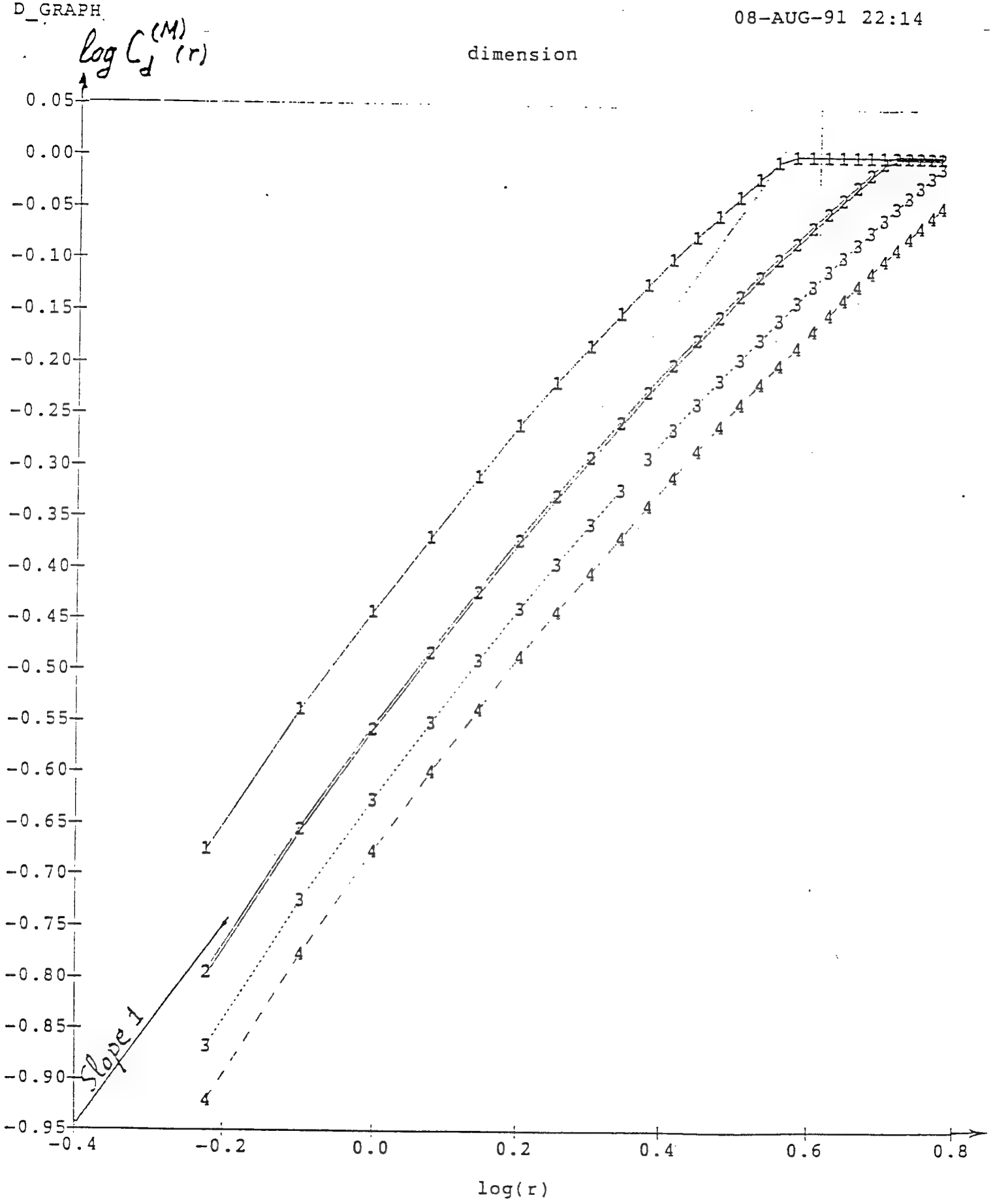
where $\alpha=1/137$ is Sommerfeld's fine structure constant and $\alpha A = (2\alpha/3\pi)(\Delta v/c)^2$ is the bremsstrahlung coefficient, or infrared exponent, of the process which generates the current j, Δv being the velocity change of the charged particles in the process. Repeating the fluctuations periodically outside the observational interval T, we can represent Eq. (81) through an equivalent Fourier series

$$\delta j/<j> = 2\alpha A \sum_{n=1}^{M} \cos(n\varepsilon_0 t+\gamma_n)/(n\varepsilon_0)^{1/2} = x(t) \qquad (82)$$

which is similar to Eq. (80), with the terms of the same frequency grouped together, and the terms lumped together in harmonics with random phases γ_n, with $M=\Lambda/\varepsilon_0$. To this expression we have applied the Takens-Grassberger-Procaccia analysis [31], by creating a time series $x_i=x(t_i)$ with $1<i<N$. We consider this series as a one-dimensional sample of the quantum 1/f process. We calculate the correlation function $C_1^{(M)}(r)$ which is N^{-2} times the number of data pairs (x_i,x_j) separated by a distance less than r. Next we form groups of d consecutive data $(x_i,x_{i+1},...x_{i+d-1})$. Considering them as vectors in a d-dimensional Euclidean space, we again calculate

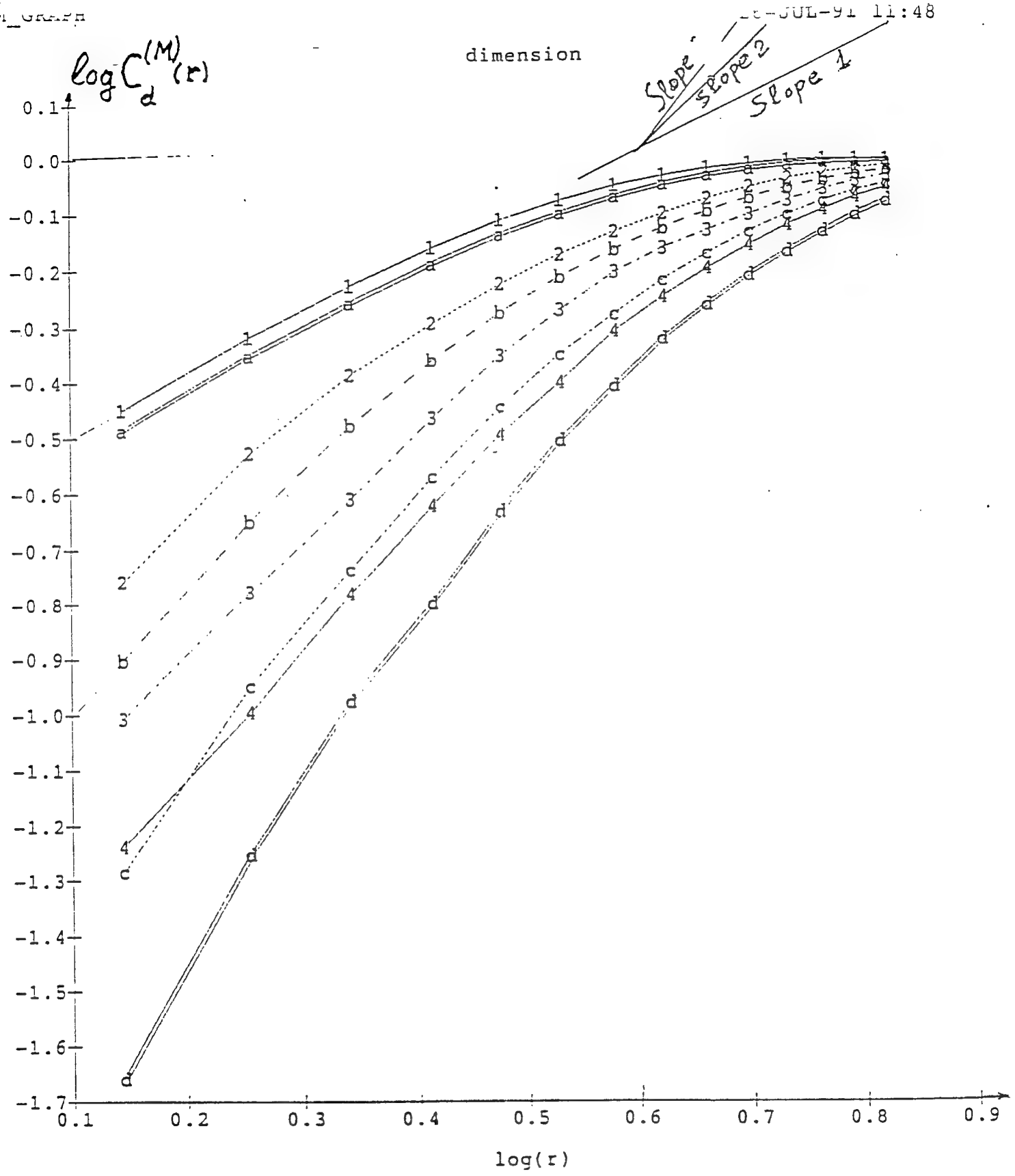
$$C_d^{(M)}(r) = N^{-2} \sum_{i;j=1}^{N} \theta[r-||x_i-x_j||], \qquad (83)$$

where $\theta(x)$ is the step function (zero for negative, 1 for positive, and 1/2 for null arguments) Finally we plot the curves $\log_{10}C_d^{(M)}(r)$ as a function of r, with d=0, 1, 2,... as a parameter. We notice (Fig. 1 for M=1 and Fig. 2 for M=2 and 3 where for M=2 the numbers d=1,2,3,4, have been replaced by a,b,c,d.) that the slope of the curves is just d, and increases as d is increased from curve to curve up to M, the number of terms in Eq. (82). We graphed the case M=1 (a single term in Eq. (82) in Fig. 1 and M=2 and 3 in



—1— d=1,deltat=0.05,E0=0.333,N=1440,M=1,deltar=0.2,NM=28
=2= d=2,L=360
·····3····· d=3,L=240
- -4- - d=4,L=180

Fig. 1



—1— d=1,E0=0.349,N=720,M=2,deltar=0.4,NM=14
═a═ M=3
·····2····· d=2,L=360,M=3
- -b- - M=3
-··3··- d=3,L=240,M=2
-···c···- M=3
—4— d=4,L=180,M=2
═d═ M=3

Fig. 2

Fig. 2. After d=M, the slope does no longer increase. This maximal slope gives the fractal dimension of the quantum 1/f process with M terms. Our conclusion at this point is that quantum 1/f noise in a frequency interval corresponding to M terms in Eq. (83) is chaotic with a fractal dimension M. Had it been stochastic rather than chaotic, the slope of the curves would have kept increasing indefinitely.

Experimentally, Fote, Kohn, Fletcher and McDonough found a fractal dimension of $d_M=10$ for 1/f noise measured in HgCdTe MW infrared detectors in the interval from $f_o=10^{-2}$Hz to F=10 Hz. Noticing that $F/f_o=10^3=2^{10}$, we expect M=10, which yields also $d_M=10$. This nice agreement between theory and experiment indicates that the measured 1/f noise is a form of (deterministic) chaos, whereas the other models which competed with the quantum 1/f theory incorrectly described it as a stochastic phenomenon. Furthermore, this agreement represents an independent verification of the quantum 1/f theory, independent from tests based on the predicted magnitude and spectral dependence of 1/f noise.

VI. QUANTUM 1/f FLICKER OF FREQUENCY IN QUARTZ RESONATORS:THEORY AND EXPERIMENT

VI.1. Introduction

Flicker of frequency noise is an important characteristic of quartz resonators which limits their stability and determines their utility for most applications. The 1/f contribution to frequency stability is best obtained by observing the stability over a range of measurement times in the time domain, or over a range of frequencies in the frequency domain. From the extended data one can fit a flicker of frequency model to the data that excludes the contributions from random walk frequency modulation and from the drift present in many resonators and oscillators (Fig. 3).

One of the first who systematically studied the 1/f noise as a function of geometry, temperature and Q-factors was Gagnepain [32]. He noticed empirically that the 1/f part of the spectral density of fractional frequency fluctuations, $S_y(f)$, varied as Q^{-4} for resonators between 1 and 25 MHz. As the temperature changes, the Q-factor of a resonator changes. This allows us to exclude the effect of many other factors. Additional work by Parker, however, showed that the data from both bulk acoustic wave (BAW) and surface acoustic wave (SAW)

devices could be roughly fit to the the same model [33], if one assumes a Q^{-4} dependence for the phase noise S_ϕ insted of S_y. The fit is shown on Fig. 4.

From a theoretical point of view, fundamental work by this author [34], inspired by the quantum 1/f theory [2], has derived the Q^{-4} depenence of S_y for all resonators from first principles already 1979. Work on many systems other than quartz has yielded very good quantitative agreement between theory and experimental data for quantum 1/f noise [35]. This paper refines the previous theoretical work on 1/f frequency noise in quartz to suggest a better framework for predicting the level of 1/f frequency noise in quartz resonators over a wide range of frequencies and Q-factors.

VI. 2. Theory of 1/f Frequency Noise in Quartz Resonators

According to the general quantum 1/f formula [2], $\Gamma^{-2}S_\Gamma(f)=2\alpha A/f$ with $\alpha=e^2/\hbar c$ $=1/137$ and $A=2(\Delta J/ec)^2/3\pi$ is the quantum 1/f effect in any physical process rate Γ. Setting $J=dP/dt=\dot{P}$ where P is the vector of the dipole moment of the quartz crystal, we obtain for the fluctuations in the rate Γ of phonon removal from the main resonator oscillation mode (by scattering on a phonon from any other mode of average frequency $<\omega>$) of the crystal, (or via a two-phonon-process at a crystall defect or impurity, involving a phonon of average frequency $<\omega'>$) the spectral density

$$S_\Gamma(f) = \Gamma^2 4\alpha(\Delta\dot{P})^2/3\pi e^2c^2, \qquad (84)$$

where $(\Delta\dot{P})^2$ is the square of the dipole moment rate change associated with the process causing the removal of a phonon from the main oscillator mode. To calculate it, we write the energy W of the interacting resonator mode $<\omega>$ in the form

$$W = n\hbar<\omega> = 2(Nm/2)(dx/dt)^2=(Nm/e^2)(e\ dx/dt)^2=(m/Ne^2)\varepsilon^2(\dot{P})^2; \qquad (85)$$

The factor two includes the potential energy contribution. Here m is the reduced mass of the elementary oscillating dipoles, e their charge, ε a polarization costant, and N their number in the quartz crystal. Applying a variation $\Delta n=1$ we get

$$\Delta n/n = 2|\Delta\dot{\mathbf{P}}|/|\dot{\mathbf{P}}|, \text{ or } \Delta\dot{\mathbf{P}}=\dot{\mathbf{P}}/2n. \quad (86)$$

Solving Eq. (85) for $\dot{\mathbf{P}}$ and substituting, we obtain

$$|\Delta\dot{\mathbf{P}}| = (N\hbar<\omega>/n)^{1/2}(e/2\varepsilon) \quad (87)$$

Substituting $\Delta\dot{\mathbf{P}}$ into Eq. (84), we get

$$\Gamma^{-2}S_\Gamma(f) = N\alpha\hbar<\omega>/3n\pi mc^2f\varepsilon^2 \equiv \Lambda/f\ . \quad (88)$$

This result is applicable to the fluctuations in the loss rate Γ of the quartz.
The corresponding resonance frequency fluctuations of the quartz resonator are given by[2]

$$\omega^{-2}S_\omega(f) = (1/4Q)(\Lambda/f) = N\alpha\hbar<\omega>/12n\pi mc^2f\varepsilon^2Q^4, \quad (89)$$

where Q is the quality factor of the single-mode quartz resonator considered, and $<\omega>$ is not the circular frequency of the main resonator mode, ω_o, but rather the practically constant frequency of the average interacting phonon, considering both three-phonon and two-phonon processes. The corresponding $\Delta\dot{\mathbf{P}}$ in the main resonator mode has to be also included in principle, but is negligible because of the very large number of phonons present in the main resonator mode.

Eq. (89) can be written in the form

$$S(f) = \beta V/fQ^4, \quad (90)$$

where, with an intermediary value $<\omega>=10^8$/s, with $n=kT/\hbar<\omega>$, T=300K and $kT=4\ 10^{14}$,

$$\beta = (N/V)\alpha\hbar<\omega>/12n\pi\varepsilon^2mc^2 = 10^{22}(1/137)(10^{-27}10^8)^2/12kT\pi10^{-27}\ 9\ 10^{20} = 1. \quad (91)$$

The form of Eq. (30) shows that the level of 1/f frequency noise depends not only as Q^{-4} as previously proposed, but also on the oscillation frequency or the volume of the active region. This model qualitatively fits the data of Gagnepain [32,42] where he varied the Q-factor with

temperature in the same resonator (but not frequency or volume).

The model also provides the basis for predicting how to improve the 1/f level of resonators, beyond just improving the Q-factor, which has been known for many years. Since the level depends on active volume, one should use the lowest overtone and smallest diameter consistent with other circuit parameters. The following experimental discussion has been generated by F. Walls (NIST Boulder).

VI. 3. Experimental Measurements and Analysis of 1/f Noise in Quartz Resonators

The level of 1/f frequency noise in quartz resonators has been measured using phase bridges and complete oscillators [32,33,36-41]. Unfortunately much of the data in the literature is unusable for modeling because the unloaded Q-factor is unknown. (Our case is even more restrictive because we also need to know the electrode size). The phase bridge approach has the advantage that the unloaded Q-factor can be easily measured and the noise in the measurement electronics can be evaluated independent of the resonator. If resonator pairs are used, the noise of the driving source can generally be neglected and the pair can operate at

Phase Noise Model for 5 MHz Oscillator

$$\mathcal{L}(f) = 10^{-12.86/f^3} + 10^{-15.0/f} + 10^{-178.7}$$

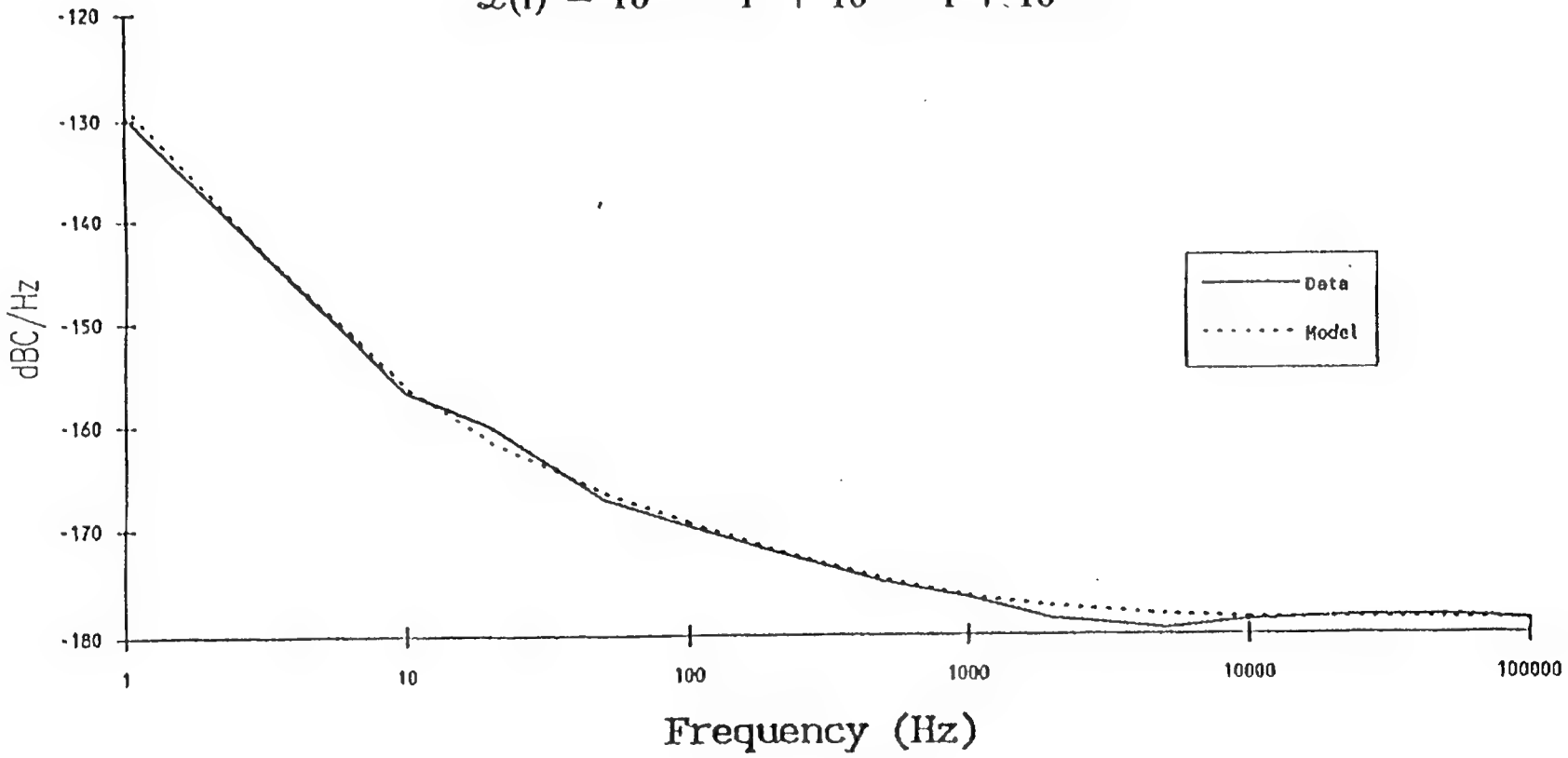


Fig. 3

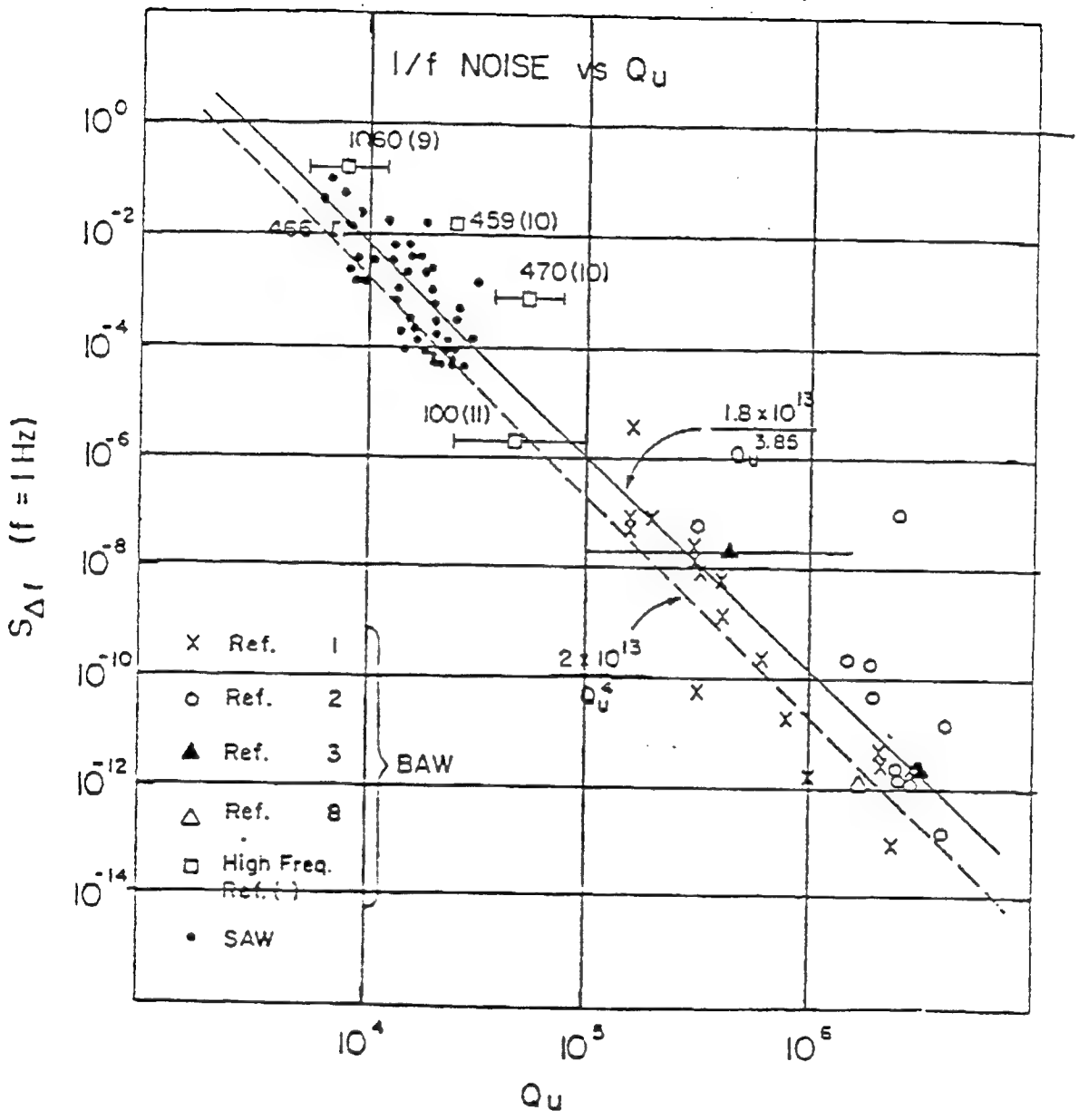


Figure 4: 1/f Noise Level at 1 Hz of Quartz Acoustic Resonators as a Function of Unloaded Q.

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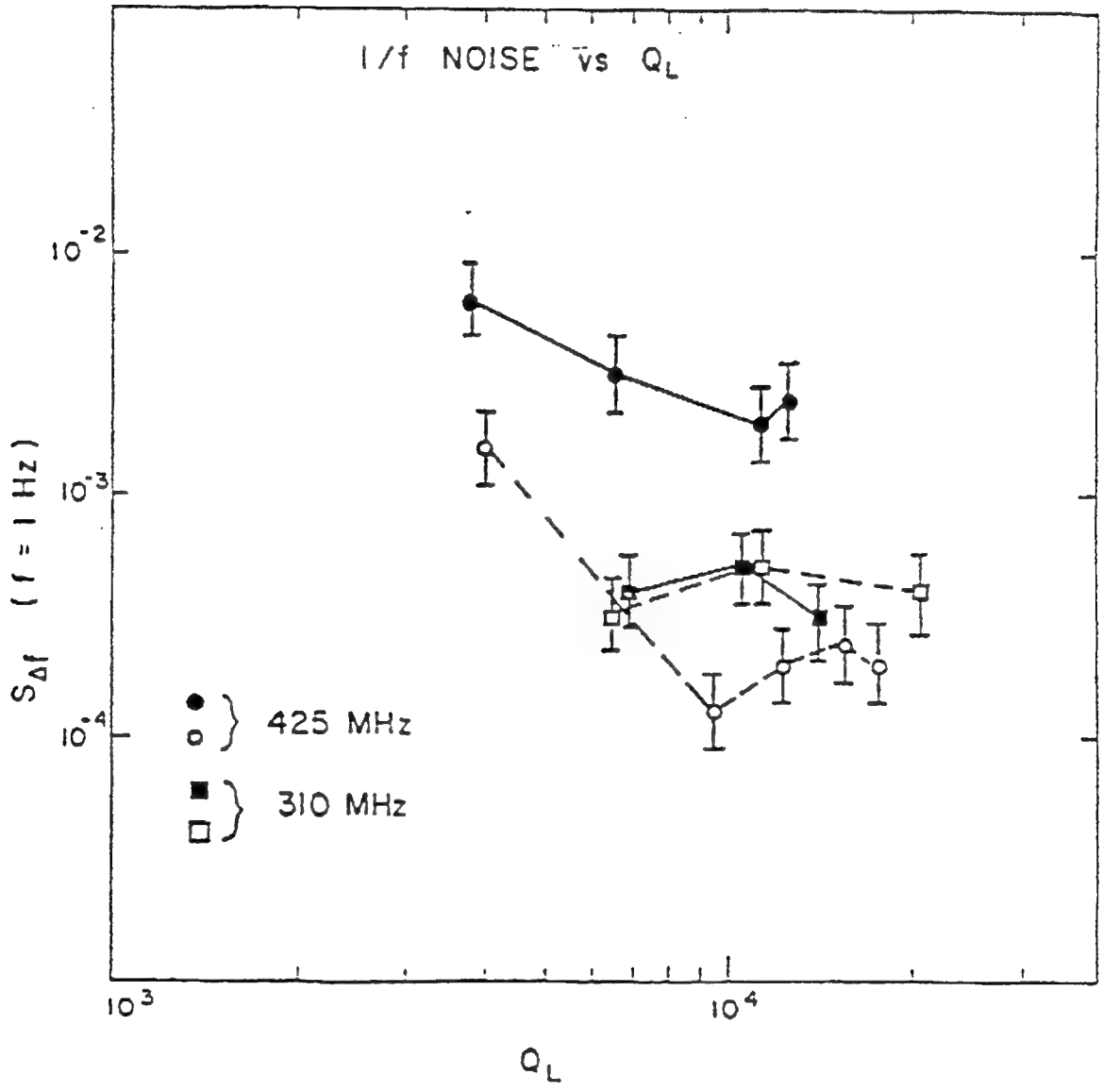
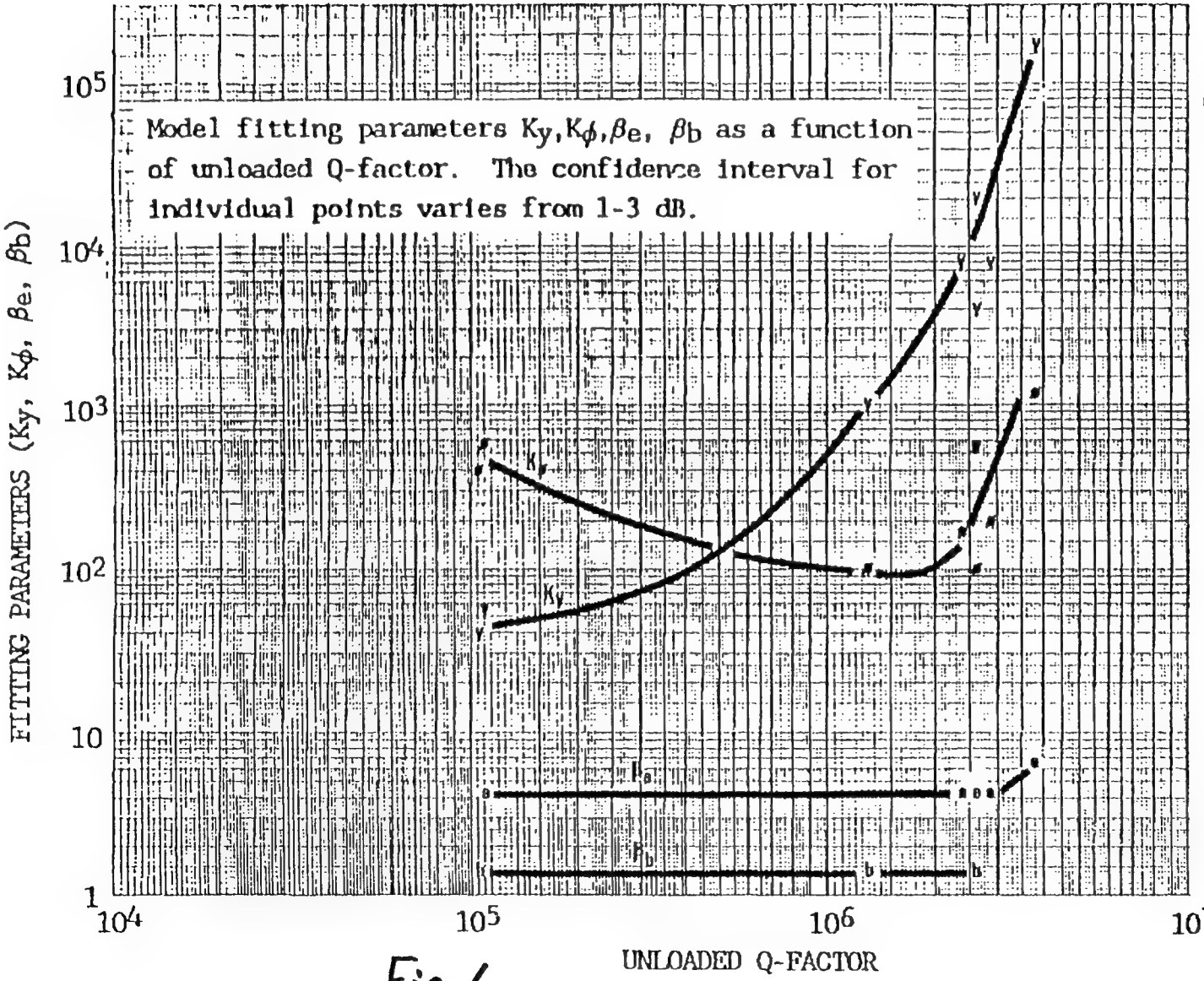


Figure 5 : 1/f Noise Level at 1 Hz of Four SAW Resonators as a Function of Loaded Q.



$$S_y = \frac{K_y}{f}\,\frac{3\times10^{-5}}{Q^4}$$

$$S_\phi = \frac{K_\phi}{f}\,\frac{3\times10^{10}}{Q^4}$$

$$S_y = \frac{\beta\, Vol(cm^3)}{Q^4}$$

Fig. 6

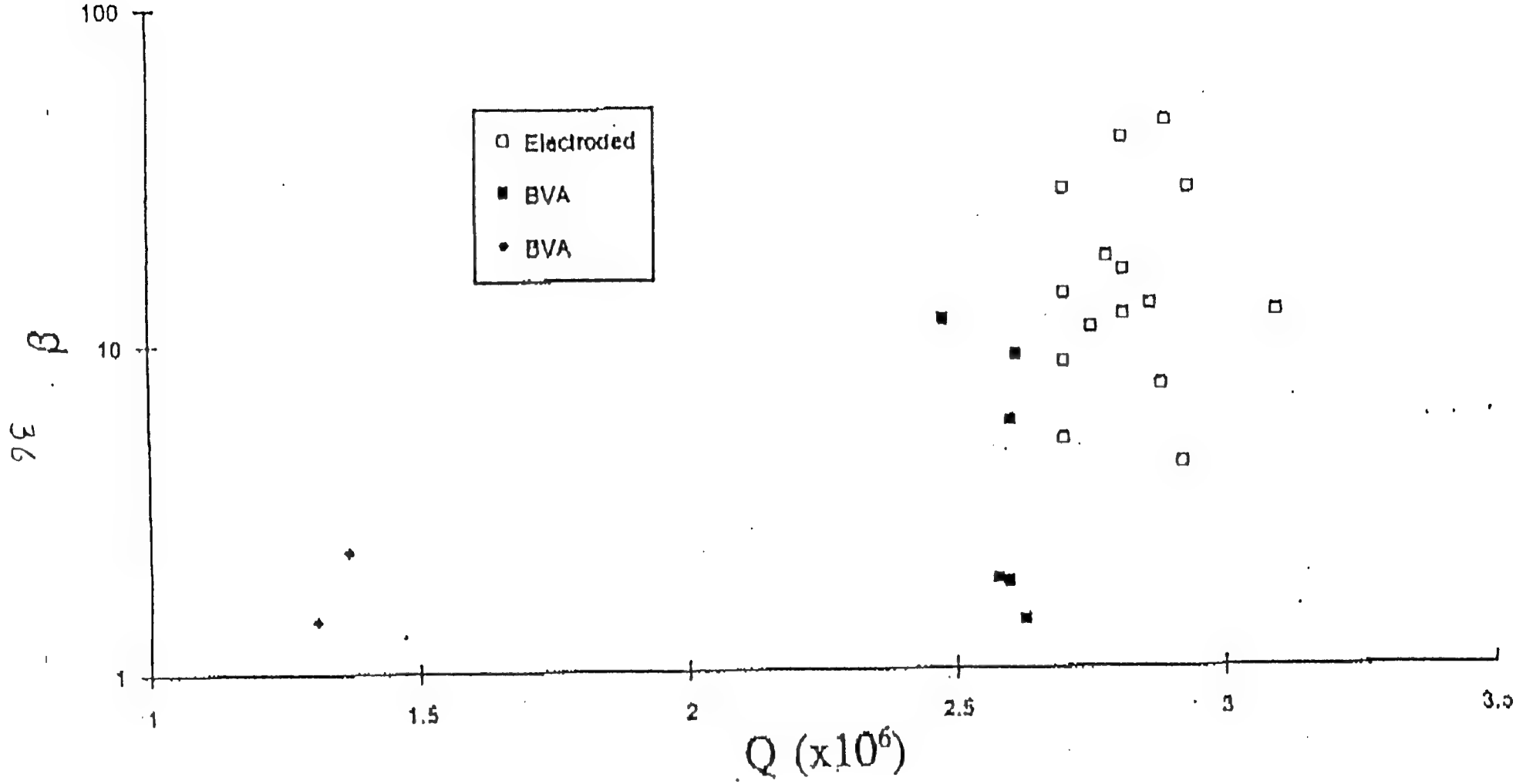


Figure 7: Fitting parameter β as a function of unloaded Q-factor for three types of resonators. The resonators in each group were matched in all known electrical parameters except Q-factor and 1/f noise [51].

virtually any frequency [39]. The oscillator approach makes it possible to compare many different resonators one at a time. The noise of individual oscillators can be derived by measuring the phase noise between 3 oscillators [42].

Figure 5 taken from [33] is one of several studies showing that the 1/f level is virtually independent of the loaded Q-factor. This is in complete agreement with the theoretical model. In practical oscillators there is a dependence on loaded Q-factor only when the phase noise of the sustaining electronics contributes to the overall noise level.

We have analyzed 1/f frequency noise as a function of unloaded Q, volume under the electrodes, and frequency. For a given resonator geometry and manufacturer, we have taken the best values of $S_y(f)$ reported in an attempt to remove the effects of poor crystals or electronics. In Fig. 6 we have taken all of the precision data available with unloaded Q-factor, electrode volume, and frequency stability and plotted them according to the three models. Except for the 2.5 MHz resonator where $Q\nu_o = 0.95\times10^{13}$, the $Q\nu_o$ product for all resonators is near 1.2×10^{13} (this is close to the material limit for AT and SC cut resonators). The curve labeled K_y shows the fit of the data to the model [32]
$S_y(f) = K_y / f(3 \times 10^{-5} / Q^4)$. K_y varies about a factor of 500 for Q-factors between 10^5 and 3.8×10^6 (resonator frequencies between 2.5 and 100 MHz).

The curve labeled K_ϕ shows the fit of the same resonator data to the model [33]
$S(f) = K_\phi/f^3$ ($3 \times 10^{10} / Q^4$). K_ϕ varies about a factor of 10 for the same range in Q-factor. Curves β_e and β_b show the fit of the same resonator data to the model $S_y(f) = \beta/f$ (Vol/Q^4), where β_e is for SC and AT resonators with electrodes plated on the resonator and β_b is for BVA-style AT and SC resonators [37]. The volume between the electrodes (in cm^3) is used to approximate the volume of quartz contributing to the output power. The β factors are remarkably constant for Q-factors from 10^5 to 3.8×10^6.

Figure 7 shows the dependence of β_e on Q-factor for a number of electroded resonators of the same type from a single manufacturer for 3 resonator types as measured by Norton [38,45]. The wide variation in β_e for the same style resonator and Q-factor indicates that acoustic loss is not the only mechanism contributing to the noise level. The data for this graph was taken from measurements of (100 s) and may have been biased high by random walk FM noise in some resonators.

VI. 4. Discussion

The analysis of the most stable quartz resonators indicates that the 1/f frequency noise level depends on volume between the electrodes and unloaded Q-factor in relatively good agreement with eq. (30) considering the fact that the estimation of $\langle\omega\rangle$ and N is only

approximative. The nonadjustable parameters β_e and β_b are virtually constant versus unloaded Q-factor, which is in stark contrast to fitting parameters K_ϕ and K_y. It is not surprising to us that β_e and β_b are different for the two types of resonators, since energy trapping and electrode stress are considerably different. Fig. 7 shows that there are other noise processes besides acoustic losses that affect the 1/f noise level in some resonators.

Although we have analyzed only the data for a few resonators, the consistency of β_e and β_b over a factor of 40 in Q-factor and resonator frequency and the general agreement for the magnitude of β between theory and experiment give us some confidence that this new model can be used to predict the best performance of different resonator geometries.

This new volume model predicts that a resonator having smaller electrodes would have a lower level of 1/f frequency noise than another one with the same frequency and Q-factor but larger diameter electrodes. The decrease in electrode area would increase the impedance levels and degrade the wide-band noise somewhat. For most resonators the wideband noise is dominated by the electronics and not the resonator. The increase in series resistance, obtained by decreasing the electrode area by a factor of 4, would probably be tolerable from the standpoint of wideband noise but might require a change in loop gain.

BT resonators are potentially interesting in that they offer a Qv_o product approximately three times higher than that of AT and SC resonators. BT cuts are roughly as sensitive to temperature transients as AT cuts. Therefore to achieve parts in 10^{-14} frequency stability with BT cuts would require temperature stabilities of order 10^{-9} K/s or 100 times better than is required for SC cut resonators [45].

Based on these early observations it appears that the level of 1/f frequency noise in quartz may yet be improved to the low 10^{-14} level by applying one or more of the following techniques: reducing the electrode area, using BVA type resonators, going to lower frequencies, using BT cut resonators. It must be remembered that acceleration induced effects become more dominant as the stability improves.

VII. EXPERIMENTAL CHECKS ON COLLECTOR QUANTUM 1/f NOISE IN BJT'S

VII.1 Introduction

1/f noise in bipolar junction transistors (BJTs) was treated by van der Ziel [46]-[48] who applied a Hooge-type approach similar to Kleinpenning's treatment [49] of pn junctions, and used experimental data to determine the Hooge constant which was in turn compared with the quantum 1/f theory. However, since the BJT is a minority carrier device, it requires the

application of the quantum 1/f equation [1,2] from the beginning, for the correct interpretation of the number of carriers in the denominator of the Langevin noise source.

In the most elementary model [50] of a BJT, the collector current I_C arises from minority carriers injected from the emitter into the base, which diffuse across the width X_B of the base and are then all swept across the reverse-biased collector junction by the built-in field of the junction. If we neglect the usually small leakage current of the collector junction and the small fraction of the carriers recombining in the base, we get for a n^+pn BJT:

$$I_C = AqD_n[n_{oB}(\exp(qV_{BE}/kT))/X_B], \qquad (92)$$

where A is the cross sectional area of the base, q=-e is the charge of the minority carriers in the base, D_n their diffusion coefficient in the base, $n_B(0)=n_{oB}\exp(qV_{BE}/kT)$ is the electron concentration at the limit of the emitter space charge region, V_{BE} is the applied base - emitter voltage, and X_B is the width of the base. The expression in rectangular brackets is the electron concentration gradient calculated with the boundary condition of a vanishing electron concentration at the limit of the collector space charge region. We assume the base to be much narrower than the electron
diffusion length $L_n=(D_n\tau)$, $X_B<<L_n$, but sufficiently wide to avoid ballistic electron transport across the base. Usually X_B is a fraction of a micron.

Quantum 1/f fluctuations of the collisional cross sections of the electrons in the base will yield fluctuations of the diffusion constant, and of the mobility ($\delta D_n/D_n=\delta\mu/\mu$):

$$\delta I_C = Aq(\delta D_n)[n_{oB}\exp(qV_{BE}/kT)/X_B]. \qquad (93)$$

The corresponding spectral density of fractional fluctuations $I^{-2}S_{IC}$ is

$$I_C^{-2}\langle(\delta I_C)^2\rangle_f = D_n^{-2}\langle(\delta D_n)^2\rangle = \mu^{-2}\langle(\delta\mu)^2\rangle = \alpha_n/fN. \qquad (94)$$

In the last step our quantum 1/f equation [1,2] was used, where N is the number of carriers which define the scattered (or diffused) current, leaving the base and emerging in the collector, while $\alpha_n = \alpha A_n$ is the effective quantum 1/f noise coefficient, or Hooge constant. The number of electrons N is thus determined by the effective lifetime τ_C of the electrons, which will be slightly lower than the lifetime in the unbounded collector material, due to the collector lead contact processes, and due to lateral surface recombination. Indeed, we can write $N = \tau_C I_C/q$. Thus we finally obtain the spectral density of the collector current fluctuations:

$$S_{Ic} = \alpha_n I_c q/(f\tau_c), \qquad (95)$$

in which τ_c is the effective lifetime of the majority carriers in the collector. This expression is simpler, but similar to the expression derived earlier, with the important difference that now we have a lifetime of the carriers in the denominator, while before it was the usually much smaller diffusion time $\tau_d = X_B^2/D_n$ of the electrons in the base. Eq. (95) also implies that in narrow-base BJTs of various base-widths α_n will be constant, as in other devices, rather than α_n/τ_d. In the following section we show that this expression is in good agreement with the experimental data in BJTs with 1/f collector noise spectra.

VII.2 Comparison of the Calculated Results with Experimental Data

1. The effective quantum 1/f noise coefficient, or Hooge constant, α_n

We consider the following scattering processes [51] in the calculation of the quantum 1/f noise coefficient α_H:

a) Normal collision processes (Impurity scattering, Optical scattering and Acoustical phonon scattering) [10].

b) Intervalley scattering; there are two types intervalley processes, i.e., g-processes which include Umklapp, and f-processes [10'].

From these points of view, we obtain the current spectral density in the form of Eq. (95):

$$S_{Ic}(f) = \alpha_n[I_c q/(f\tau_c)] = \alpha A_n[I_c q/(f\tau_c)]$$

For the case a), the normal collision processes:

$$\alpha_n = \alpha A_n = \alpha 4\Delta v^2/(3\pi c^2) = 4\alpha(kT/m_o\pi c^2) = 4.69\times10^{-10}, \qquad (96)$$

where we used $\alpha = \mu_o c e^2/2h = 1/137$ (the fine structure constant), $c = 3\times10^8$m/sec, $k = 1.38\times10^{-23}$J/K, $T = 300$K, and $m_o = 9.1\times10^{-31}$kg.

For the case b), the intervalley scattering + umklapp scattering 1/f noise (g-processes):

$$\alpha_n = \alpha A_n = \alpha 4\Delta v^2/(3\pi c^2) = \alpha 4(\hbar\Delta k/m)^2/3\pi c^2 = 5.86\times10^{-7}, \qquad (97)$$

where $\Delta v = \Delta p/m = \hbar\Delta k/m$, $|\Delta k| = 0.8(2\pi/a)$, and $a = 5.4\text{Å}$ for Silicon [51]. The g-proceses include Umklapp back to the original Brillouin zone, and the conduction effective mass $m = 0.26m_o$ must be used [10'].

Comparing the $\alpha_n = 5.86 \times 10^{-7}$ with reference [10] Fig. 8 where the result was from the exact calculation, we find out that they are pretty close. We would like to point out that $\alpha_n \cong \alpha_{Intervalley}$ which is only an approximation due to the high $\alpha_{Intervalley}$ comparing with $\alpha_{impurity}$ and $\alpha_{acoustic}$.

2. The experimental data compared to the Hooge parameters α_{Hn}

i. In n^+-p-n bipolar transistors:

$$S_\mu(f)/\mu^2 = \alpha_H/fN. \tag{98}$$

This can be written in terms of the diffusion constant , $D = kT\mu/q$

$$S_D(f)/D^2 = \alpha_H/fN \tag{99}$$

which yields [52]

$$S_{Ic}(f) = \frac{\alpha_{Hn}}{f}\frac{qI_cD_n}{w_B^2}\ln[\frac{N(0)}{N(w_B)}], \tag{100}$$

where α_H is the Hooge parameter for electrons, f is the frequency, D_n is the diffusion constant for electrons, w_B is the width of the transistor base region, N(0) is the electron concentration for unit length at the emitter side of the base, and $N(w_B)$ is the electron concentration for unit length at the collector side of the base.
If we introduce the ratio [53]

$$\frac{N(0)}{N(w_B)} \le \frac{v_n + D_n/w_B}{D_n/w_B} \tag{101}$$

where v_n is the saturation velocity of the electrons in the base region, and the diffusion time

$$\tau_{dn} = w_B^2/2D_n = 1/2\pi f_T \tag{102}$$

where f_T is the upper cut off frequency of the BJT, then

$$S_{Ic}(f) = \frac{\alpha_{Hn}}{f}\frac{qI_C}{2\tau_{dn}}\ln\left[\frac{v_n+(D_n/w_B)}{D_n/w_B}\right] = \frac{\alpha_{Hn}}{f}qI_C\pi f_T\ln\left[\frac{V_n+D_n/w_B}{D_n/w_B}\right] \quad (103)$$

ii. In p^+-n-p bipolar transistors:

$$S_{Ic}(f) = \frac{\alpha_{Hp}}{f}\frac{qI_CD_p}{w_B^2}\ln\left[\frac{P(0)}{P(w_B)}\right] \quad (104)$$

where

$$\frac{P(0)}{P(w_B)} = 1 + \frac{v_{cp}w_B}{D_p}, \text{ and } \tau_{dp} = w_B^2/2D_p, \quad (105)$$

and where v_{cp} ($\approx 10^7$ cm/s in Si) is the saturation velocity of holes in the base. For details see[53], and the following comparisons in table I, where a refers to the normal scattering quantum 1/f calculation and b to the more likely case of g-type intervalley-umklapp scattering. It is more likely to have this case since it has a much larger quantum 1/f effect, and will mask the smaller contribution calculated for case a. Also, the experimentally noticed strong increase in 1/f noise if the transistors are cut from single-crystals so that the current flows along an [100] - like direction confirms this and can not be explained without the quantum 1/f theory. The experimental values in Table I are much closer to the values calculated for the case b replacing τ_{nd} with the lifetime in the collector, τ_c. This corresponds to the inclusion of a corrective factor τ_{nd}/τ_c in the derivation of the collector quantum 1/f noise spectral density. However, a proportionality of S_{Ic} to f_T is noticed in many transistors, contrary to our suggestion of replacing τ_{nd} with τ_c. The present formula containing τ_c is applicable to ultrasmall BJTs with a very narrow base region for which most of the life time of carriers diffusing from the emitter is spent in the collector. This subject requires further theoretical and experimental study to determine the exact limits of applicability of the new formula.

Table-1: The experimental data vs Hooge parameters*: (*: BJTs No.1-MRF90 has $f_T = 450MH_z$ No.2-NEC57867 has $f_T = 8GH_z$.)

Current I_c (mA)	BJTs	$S_{I_c}^{exp}$ (A^2/Hz)	τ_{dn} (sec)	α_{Hn}	τ_c (sec)	theory $S(\tau_c)$ I_c (A^2/H_z)	theory-a $S(\tau_{dn})$ I_c (A^2/H_z)	theory-b $S(\tau_{dn})$ I_c (A^2/H_z)
0.1	No. 1	9.00E-23	3.50E-10	5.80E-09	1.0E-7		2.15E-23	2.68E-20
	No. 2	1.10E-22	2.00E-11	8.30E-10	1.0E-7		3.76E-25	4.69E-19
						a: 7.51E-26		
						b: 9.39E-23		
0.2	No. 1	2.20E-22	3.50E-10	7.00E-09	1.0E-7		4.29E-23	5.37E-20
	No. 2	3.10E-22	2.00E-11	1.20E-09	1.0E-7		7.51E-25	9.39E-19
						a: 1.50E-25		
						b: 1.88E-22		
0.3	No. 1	4.60E-22	3.50E-10	9.80E-09	1.0E-7		6.44E-23	8.05E-20
	No. 2	7.50E-22	2.00E-11	1.90E-09	1.0E-7		1.13E-24	1.41E-18
						a: 2.25E-25		
						b: 2.82E-22		
0.4	No. 1	6.60E-22	3.50E-10	1.30E-08	1.0E-7		8.59E-23	1.07E-19
	No. 2	1.60E-21	2.00E-11	3.00E-09	1.0E-7		1.50E-24	1.88E-18
						a: 3.00E-25		
						b: 3.76E-22		
0.5	No. 1	1.50E-21	3.50E-10	1.90E-08	1.0E-7		1.07E-22	1.34E-18
	No. 2	3.50E-21	2.00E-11	5.30E-09	1.0E-7		1.88E-24	2.35E-18
						a: 3.76E-25		
						b: 4.69E-22		

VIII. QUANTUM 1/f MOBILITY FLUCTUATIONS IN SEMICONDUCTORS, CALCULATED WITH THE QUANTUM 1/f CROSS-CORRELATION FORMULA

VIII. 1. Introduction

Together with the graduate student Thomas H. Chung, the author has performed an analytical calculation of mobility fluctuations in silicon and gallium arsenide, using the new quantum 1/f cross-correlations formula derived by the author in the previous AFOSR Grant period, (1987) and included in the July 1989 Final Technical Report. This same new form of the quantum 1/f cross correlations was rederived with a different method by Van Vliet in 1989. It differs from the old form used in the 1985 calculation of Kousik et. al. by a correction which is zero when the momentum changes of the two current carriers involved in the cross correlation are identical, but increases to finite values when the momentum differences caused by the scattering process are different. The correction is proportional to the squared difference of the two momentum changes. We have repeated all calculations in the original paper by Kousik et.al. [10, 10'], obtaining both for impurity scattering and for the various types of phonon scattering new analytical expressions which show a considerable increase of the final quantum 1/f noise. The results obtained are applicable both to direct and indirect bandgap semiconductors.This calculation is of major importance for the 1/f noise-related optimization both of the two types of materials, and of the many devices constructed with them for military and civilian applications in the electronic and opto-electronic industry.

The new cross-correlation formula gives the cross-spectral density which describes the way in which symultaneous quantum 1/f scattering rate fluctuations ΔW obseved in the direction of the outgoing scattered wave-vector K' are correlated with those in the K" direction, when the two corresponding incoming current carriers have the wave vectors K_1 and K_2:

$$S_{\Delta W}(K_1,K';K_2,K'';f) = (2\alpha/3\pi f)(\hbar/m^*c)^2 W_{K_1,K'} \cdot W_{K_2,K''}[(K'-K_1)^2+(K''-K_2)^2]\delta_{K_1,K_2}. \qquad (106)$$

The form conjectured by us earlier had $2(K'-K_1)(K''-K_2)$ in place of the rectangular bracket. The difference between the rectangular bracket and $2(K'-K_1)(K''-K_2)$ is the perfect square $[(K'-K_1)-(K''-K_2)]^2$. Therefore we expect the new results to be always larger than the results obtained on the basis of the previously conjectured form.

VIII. 2. Impurity Scattering

For impurity scattering of electrons in solids, fluctuations $\Delta\tau$ of the collision times τ will cause mobility fluctuations

$$\Delta\mu_{band}(t) = [e/m^*<<v^2>>]\Sigma_K v_K^2 \Delta\tau(t) n_K, \qquad (107)$$

where $<<v^2>>$ is both the average over all states of wave-vectors **K**, with occupation numbers n_K, in the conduction band, and the thermal equilibrium average of the quadratic carrier velocities. With the help of the relation

$$1/\tau(K) = (V/8\pi^3)\int(1 - \cos\theta'/\cos\theta) W_{K,K'} d^3K', \qquad (108)$$

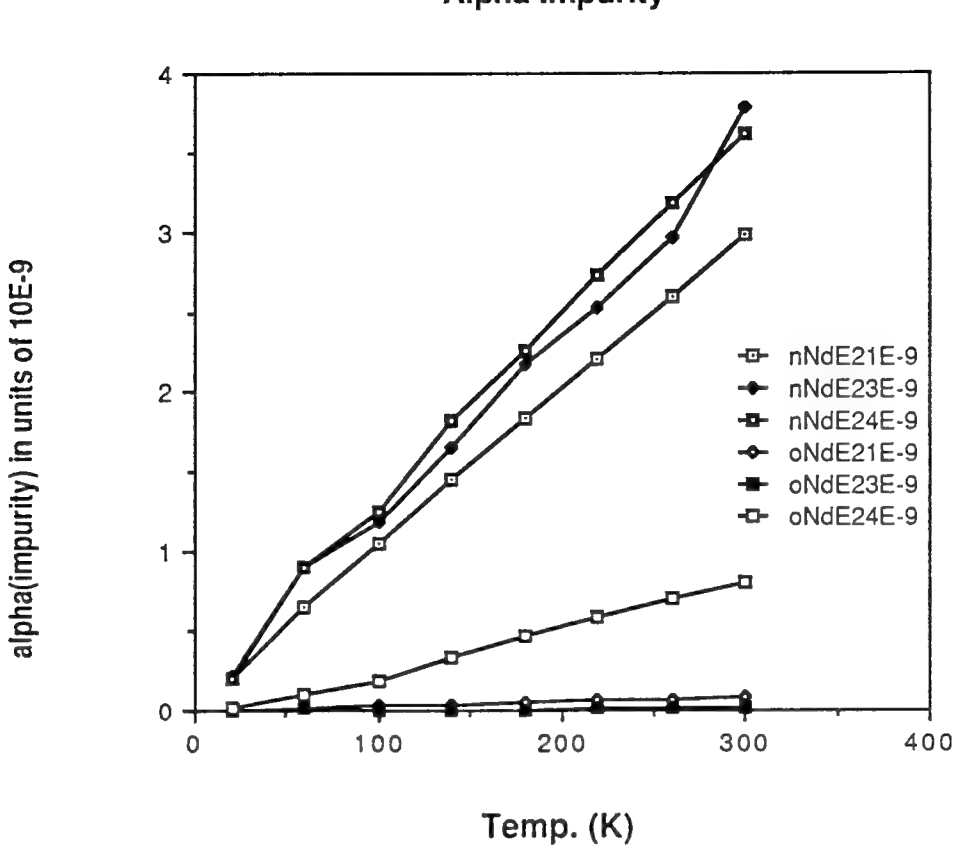
the mobility fluctuations are reduced to fluctuations of the elementary scattering rates $W_{K,K'}$, governed by Eq. (106). Here V is the volume of the normalization box which disappears in the final result, θ and θ' respectively the angles **K** and **K'** form with the direction of the applied field. One finally obtains after tedious multiple integrations

$$\mu^{-2} S_{\Delta\mu}(f) = [256\pi\alpha\kappa^2\varepsilon^4\hbar^{12}/3m^{*8}Z^4e^8N_i^2](1/f)\Sigma_K K^{10}[\ln(1+a^2)-a^2/(1+a^2)]^{-3}$$
$$[(2a^2+a^4)/(1+a^2)-2\ln(1+a^2)]F(E_K)[\Sigma_K v_K^2 \tau(K) F(E_K)]^{-2}, \qquad (109)$$

where $a=2K/\kappa$, $\kappa^2=e^2n(T)/\varepsilon k_B T$, $n(T)$ is the electron concentration, $F(E_K)=\exp(E_F-E_K)$ for non-degenerate semiconductors, N_i the concentration of impurities of charge Ze and ε the dielectric constant. The corresponding partial Hooge parameter for impurity scattering is thus

$$\alpha_i = [4\sqrt{2}\pi\alpha\kappa\hbar^5 N_c/3m^{*7/2}(k_B T)^{3/2}c^2]\int_0^\infty dx\, x^{11/2}e^{-x}$$
$$[\ln(bx+1)-bx/(bx+1)]^{-3}[(2bx+b^2x^2)/(bx+1)-2\ln(bx+1)]$$
$$\{\int_0^\infty dx\, x^3 e^{-x}[\ln(bx+1)-bx/(bx+1)]^{-1}\}^{-2}. \qquad (110)$$

This result is graphed below for three different values of the donor concentration N_d (=10^{21}, 10^{23}, and 10^{24}) and is compared with old results (marked with an initial letter o)obtained by simply recalculating the old analytical expression [10].



As expected, the new cross correlation formula leads to higher α_i values than the previously conjectured expression. This was mentioned in connection with Eq. (106) above.

VIII. 3. Acoustic Electron-Phonon Scattering

In this case the calculation is similar, and leads to the result

$$\alpha_{ac} = [32\pi\alpha N_c m^* C^7 \hbar^3/3c^2 k_B T)^4]\{(1/R^2)\int_1^\infty dx x^{-4}$$
$$[(x-1)^7/7+(R+1)(x-1)^6/6+R(x-1)^5/5]$$
$$[(x-1)^5/5+(R+1)(x-1)^4/4+R(x-1)^3/3]\exp(-x^2/4R)$$
$$+\int_0^1 dx x^{-4}[(x+1)^5/5-(x+1)^6/6+(x-1)^5/5+(x-1)^6/6]$$
$$[(x+1)^3/3+(x-1)^4/4+(x-1)^3/3-(x+1)^4/4]\exp(-x^2/4R)$$

$$+\int_1^\infty dx\, x^{-4}[(x+1)^5/5-(x+1)^6/6][(x+1)^3/3-(x+1)^4/4]\exp(-x^2/4R)\}, \qquad (111)$$

where $R=k_BT/2m^*C_1^2$, C_1 is the deformation potential, and N_c is the effective density of states for the conduction band.

VIII. 4. Non-Polar Optical Phonon Scattering

This time one obtains

$$\alpha_{n.o.ph} = [8\pi\sqrt{2\hbar\omega_o\alpha}N_c\hbar^2/3m^{*5/2}c^2\omega_o]\{\int_0^\infty dx\, x^{5/2}$$
$$[(F+1)(x-1)^{1/2}\theta(x-1)+F(x+1)^{1/2}]^{-4}$$
$$[(F+1)^2(x-1)(2x-1)\theta(x-1)+F^2(x+1)(2x+1)]\exp(-\hbar\omega_o x/k_BT)\}$$
$$\{\int_0^\infty dx\, x^{3/2}[(F+1)(x-1)^{1/2}\theta(x-1)+F(x+1)^{1/2}]^{-1}\exp(-\hbar\omega_o x/k_BT)\}^{-2}, \qquad (112)$$

where $F=[\exp(\hbar\omega_o/k_BT)-1]^{-1}$, and ω_o is the optical phonon frequency.

VIII. 5. Polar Optical Phonon Scattering

Proceeding as in Secs. VIII.2 and .4, we obtain

$$\alpha_{p.o.ph} = [8\pi\sqrt{2\hbar\omega_l\alpha}N_c\hbar^2/3m^{*5/2}c^2\omega_l]\{\int_0^\infty dx\, x^4$$
$$[F^2(x+1)^{1/2}\ln(2x^{1/2}+2(x+1)^{1/2})$$
$$+(F+1)^2(x-1)^{1/2}\ln(2x^{1/2}+(x-1)^{1/2})\theta(x-1)]\exp(-\hbar\omega_l x/k_BT)$$
$$[(F+1)\operatorname{arcsinh}(x-1)^{1/2}\theta(x-1)+F\operatorname{arcsinh}(x^{1/2})]^{-4}\}. \qquad (113)$$

Here ω_l is the longitudinal phonon frequency.

VIII. 6. Intervalley Scattering

This type of scattering, present in indirect bandgap semiconductors, transfers electrons from one of the six minima (or valleys) of the conduction band energy in k-space to one of the other five minima. Transitions between a valley and the nearest valley, which is along the same k-space direction in the next copy of the first Brilloin zone in the periodic zone scheme, are of the Umklapp type, and are called g-processes. Transitions to the four valleys present in the same zone along the other two k-space directions are called f-processes. Repeating a previous

calculation [10'] on the basis of the new cross-correlation formula (106), we obtain for g-processes

$$\alpha_g = [8\pi\sqrt{2\hbar\omega_{ij}\alpha}N_c\hbar^2/3m^{*5/2}c^2\omega_{ij}]\{\int_0^\infty dx x^{5/2}$$
$$[(F+1)(x-1)^{1/2}\theta(x-1)+F(x+1)^{1/2}]^{-4}$$
$$[(F+1)^2(x-1)(2x-1)\theta(x-1)+F^2(x+1)(2x+1)]\exp(-\hbar\omega_{ij}x/k_BT)\}$$
$$\{\int_0^\infty dx x^{3/2}[(F+1)(x-1)^{1/2}\theta(x-1)+F(x+1)^{1/2}]^{-1}\exp(-\hbar\omega_{ij}x/k_BT)\}^{-2}, \quad (114)$$

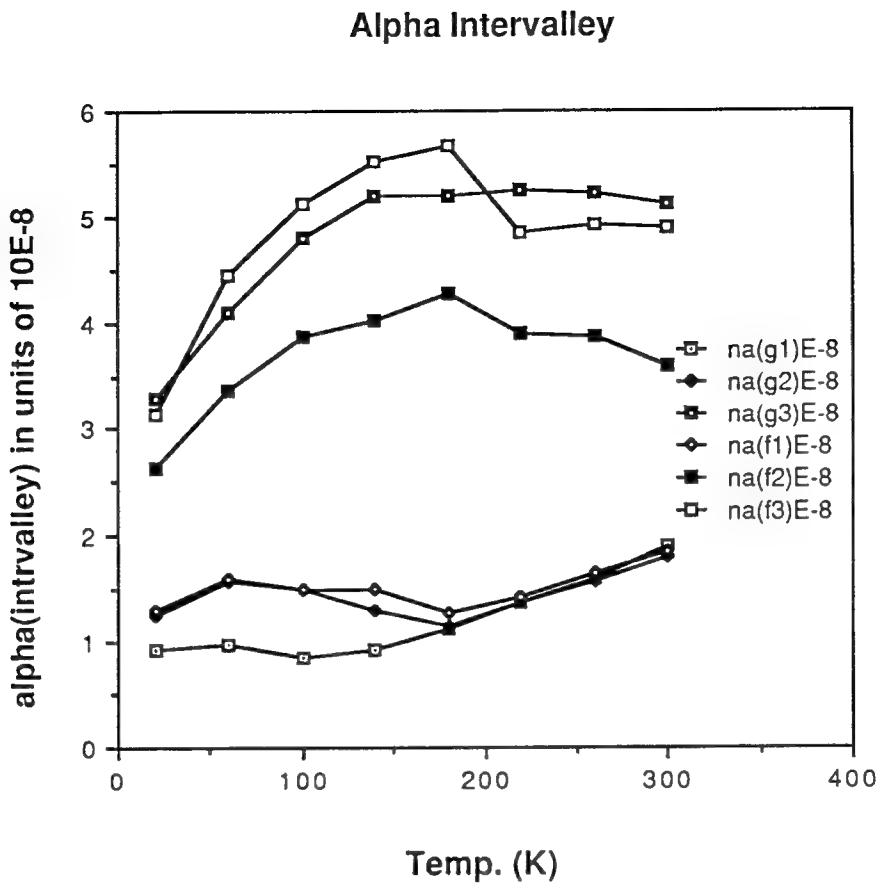
where $\hbar\omega_{ij}$ is the phonon energy corresponding to the momentum difference required by the intervalley transition. For the corresponding f-process we obtain [10']

$$\alpha_f = (k_o/q_o)^2\alpha_g, \quad (115)$$

where k_o/q_o is the ratio between the position vector of a conduction band energy minimum in k space, and twice the distance of the minimum from the Brillouin zone boundary, 0.85/0.3 for silicon. There are three g-type alphas α_{g1}, α_{g2} and α_{g3} (from LA, TA and LO phonons repectively) and three f-type contributions α_{f1}, α_{f2} and α_{f3} (from TA, LA and TO phonons). Their values are given in the graph below and are a few times larger than the old values.

The various quantum 1/f contributions derived here can be approximately superposed to yield the resultant quantum 1/f coefficient according to the rule

$$\alpha_H = \Sigma_i(\mu/\mu_i)^2\,\alpha_i \quad (116)$$



IX. COHERENT QUANTUM 1/f CHAOS

Conventional quantum 1/f fluctuations of physical cross sections and process rates have been introduced by us as a fundamental infrared divergence phenomenon in 1975 [1]. Some of the subsequent publications [2]-[13] have shown this new effect to be unaffected by the presence of the thermal radiation background [4], [5], some have derived it with wave packets [11], including a finite mean free path [9], in second quantization [13], with the Keldysh-Schwinger method [12], or in the Van Hove weak interaction limit [14], some derived its characteristic functional [7], or applied it to the calculation [10] of mobility and recombination speed fluctuations in semiconductors and semiconductor devices. Others verified the new effect experimentally and successfully applied it to electronic devices.

The present paper derives a related fundamental effect which we call the coherent 1/f effect, with elementary methods of quantum electrodynamics and non-relativistic many-body theory. Our derivation uses the new picture introduced by Dollard [55] and generalized by Kulish and Fadde'ev [56] and later by Zwanziger [57], in agreement with earlier work by

Chung [58] and by Kibble [59]. In this new picture, the asymptotic Coulomb interaction is included in the unperturbed Hamiltonian rather than in the perturbation part. This leads to a more complex physical free particle notion which includes a coherent photon cloud, and replaces the pole in the propagator with a branch point. It also leads to a smeared-out mass shell. Using this picture, we can neglect the remaining part of the interaction if we limit ourselves to the asymptotic region of large distances and times, which are important in the case of 1/f noise.

For N electrons in a Fermi sphere shifted in momentum space by a vector $\mathbf{p}_o$ and occupying N/2 orbitals $e^{i\mathbf{p}\mathbf{r}}$, the propagator derived by these authors [57] can be reduced for large time components of x'-x to the non-relativistic form

$$-i\langle\Phi_o|T\psi_{s'}(x')\psi_s^{\dagger}(x)|\Phi_o\rangle \equiv \delta_{ss'}\, G_s(x'-x)$$

$$= (i/V)\sum_{\mathbf{p}}\{exp\,i[\mathbf{p}(\mathbf{r}-\mathbf{r}')-\mathbf{p}^2(t-t')/2m]/\hbar\}n_{\mathbf{p},s}$$

$$\times\{-i\mathbf{p}(\mathbf{r}-\mathbf{r}')/\hbar+i(m^2c^2+\mathbf{p}^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}. \qquad (117)$$

Here $\alpha=e^2/\hbar c=1/137$ is Sommerfeld's fine structure constant, $n_{\mathbf{p},s}$ the number of electrons in the state of momentum $\mathbf{p}$ and spin s, m the rest mass of the fermions, $\delta_{ss'}$ the Kronecker symbol, c the speed of light, $x=(\mathbf{r},t)$ any space-time point and V the volume of a normalization box. T is the time-ordering operator which orders the operators in the order of decreasing times from left to right and multiplies the result by $(-1)^P$, where P is the parity of the permutation required to achieve this order. For equal times, T normal-orders the operators, i.e., for t=t' the left-hand side of Eq. (117) is $i\langle\Phi_o|\psi_s^{\dagger}(x)\psi_{s'}(x')|\Phi_o\rangle$. The state Φ_o of the N electrons is described by a Slater determinant of single-particle orbitals.

Consider first the case t=t' for simplicity, although only the case of large t-t' can be expected to be experimentally applicable. The pair correlation function can then be decomposed as follows

$$\langle\Phi_o|\psi_s^{\dagger}(x)\psi_{s'}^{\dagger}(x')\psi_{s'}(x')\psi_s(x)|\Phi_o\rangle = \langle\Phi_o|\psi_s^{\dagger}(x)\psi_s(x)|\Phi_o\rangle\langle\Phi_o|\psi_{s'}^{\dagger}(x')\psi_{s'}(x')|\Phi_o\rangle$$

$$- \langle\Phi_o|\psi_s^{\dagger}(x)\psi_{s'}(x')|\Phi_o\rangle\langle\Phi_o|\psi_{s'}^{\dagger}(x')\psi_s(x)|\Phi_o\rangle. \qquad (118)$$

The first term can be expressed in terms of the particle density of spin s, $n/2 = N/2V = \langle\Phi_o|\psi_s^{\dagger}(x)\psi_s(x)|\Phi_o\rangle$, while the second term can be expressed in terms of the Green function (117) in the form

$$A_{ss'}(x-x') \equiv\langle\Phi_o|\psi_s^{\dagger}(x)\psi_{s'}^{\dagger}(x')\psi_{s'}(x')\psi_s(x)|\Phi_o\rangle = (n/2)^2 + \delta_{ss'}\, G_s(x'-x)G_s(x-x'). \qquad (119)$$

The "relative" autocorrelation function A(x-x') describing the normalized pair correlation independent of spin is obtained by dividing by n^2 and summing over s and s'

$$A(x-x') = 1 + (1/n^2)\sum_s G_s(x-x')G_s(x'-x)$$

$$= 1 - (1/N^2)\sum_s \sum_{pp'}\{\exp[i(p-p')(r-r')/\hbar]\}n_{p,s}n_{p',s} \times |p(r-r')/\hbar|^{\alpha/\pi}|p'(r-r')/\hbar|^{\alpha/\pi}. \quad (120)$$

Here we have used Eq. (117). The low-wavenumber part A_l of this relative density autocorrelation function is given by the terms with $p=p'$.

$$A_l(x-x') = 1 - (1/N^2)\sum_s \sum_p n_{p,s}|p(r-r')/\hbar|^{2\alpha/\pi} \quad (121)$$

$$= 1 - (2/N^2)[V/(2\pi\hbar)^3] \int_{p<p_F} d^3p\,|(p+p_o)(r-r')/\hbar|^{2\alpha/\pi}$$

$$= 1 - (2/N^2)[V/(2\pi\hbar)^3] \int_{-a}^{a} dp_1 \int_{-a}^{a} dp_2 \int_{-a}^{a} dp_3\,|(p_3+p_{o3})(r-r')/\hbar|^{2\alpha/\pi}$$

$$= 1 - (2/N^2)[V/(2\pi\hbar)^3]8a^3|a(r-r')/\hbar|^{2\alpha/\pi}$$

$$= 1 - (1/N)[(\pi/6)^{1/3}p_F|r-r'|/\hbar]^{2\alpha/\pi}. \qquad \text{for } p_F>>p_{o3}; \quad (122)$$

and

$$= 1 - (1/N)|p_o(r-r')/\hbar|^{2\alpha/\pi} \qquad \text{for } p_F<<p_{o3}; \quad (123)$$

In view of the smallness of $2\alpha/\pi$, for $p_F>>p_{o3}$ we have integrated in cartesian coordinates, approximating the Fermi sphere by a cube of side 2a with $a=(\pi/6)^{1/3}p_F$. The result is practically independent of p_o and of $p_{o3}=p_o(r-r')/|r-r'|$. The factor $(\pi/6)^{2\alpha/3\pi}$ can be neglected. For $p_F<<p_{o3}$ we used the mean value theorem for estimating the integral over d^3p in spherical coordinates. In all cases the autocorrelation decreases very slowly from 1 when $|r-r'|$ is increased to very large values. Writing the rectangular bracket in Eq. (122-123) as an exponential function of its logarithm, expanding the resulting exponential, and keeping only the first term, we obtain with $p_F/\hbar=k_F$

$$A_l(x-x') = (1-2/N) + (6/\pi)^{2\alpha/3\pi}\{[1/k_F|r-r'|]^{2\alpha/\pi}\}/N$$

$$= (1-2/N) + (6/\pi)^{2\alpha/3\pi}[1/k_F]^{2\alpha/\pi}(2\alpha/N\pi)\int_0^\infty \cos[k|r-r'|]dk/k^{1-2\alpha/\pi}.$$

$$\approx\{N-2 + (2\alpha/\pi)\int_0^\infty [k/k_F]^{2\alpha/\pi}\cos[k|r-r'|]dk/k\}/N \quad \text{for } p_F>>p_{O3}; \tag{124}$$

$$\approx\{N-2 + (2\alpha/\pi)\int_0^\infty [k/k_O]^{2\alpha/\pi}\cos[kp_O(r-r')/p_O]\, dk/k\}/N \quad \text{for } p_F<<p_{O3}; \tag{125}$$

Here we have used a well-known Fourier integral [60] and we have introduced $k_O \equiv p_{O3}/\hbar$. According to the Wiener-Khintchine theorem, the coefficient of the cos gives the spectral density. To get it for the fractional fluctuations $\delta n/n$, we divide by the constant term N-2

$$S_{\delta n/n}(k) = [2\alpha/\pi k(N-2)][k/K]^{2\alpha/\pi}, \tag{126}$$

where $K=k_F$ for $p_F>>p_{O3}$ as in the case of metals with spherical wave symmetry, and $K=k_O$ for $p_F<<p_{O3}$. Although inapplicable, this pure 1/k spectrum is the wave-number equivalent of the coherent quantum 1/f noise derived earlier [61], [62] in excellent agreement with the experiments on large electronic devices [46]. Due to $2\alpha A<<1$ the second factor is practically unity and of no importance, except for eliminating the logarithmic divergence from the spectral integral. This wave number spectrum also entails a 1/f frequency spectrum obtained by writing dk/k=df/f, as was shown in detail in a previous paper [13]. For equal times our result can not be expected to be valid, due to the asymptotic character of Eq. (117). We shall now derive the 1/f spectrum directly below.

If $t\neq t'$, Eq. (118) is replaced by

$$\langle\Phi_O|T\psi_s^\dagger(x)\psi_s(x)\psi_{s'}^\dagger(x')\psi_{s'}(x')|\Phi_O\rangle = \langle\Phi_O|\psi_s^\dagger(x)\psi_s(x)|\Phi_O\rangle\langle\Phi_O|\psi_{s'}^\dagger(x')\psi_{s'}(x')|\Phi_O\rangle$$
$$- \langle\Phi_O|T\psi_{s'}(x')\psi_s^\dagger(x)|\Phi_O\rangle\langle\Phi_O|T\psi_s(x)\psi_{s'}^\dagger(x')|\Phi_O\rangle. \tag{127}$$

Eq. (119) remains the same, except for the middle part which is replaced by the left hand side of Eq. (127). Eq. (120) becomes now

$$A(x-x') = 1 - (1/n^2)\sum_s G_s(x-x')G_s(x'-x)$$

$$= 1 - (1/N^2)\sum_s \sum_{pp'}\{\exp i[(p-p')(r-r')-(p^2-p'^2)(t-t')/2m]/\hbar\}n_{p,s}n_{p',s}$$
$$\times\{p(r-r')/\hbar-(m^2c^2+p^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}$$
$$\times\{p'(r-r')/\hbar-(m^2c^2+p'^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}. \tag{128}$$

Here we have used again Eq. (117). The low-frequency and low-wavenumber part A_l of this relative density autocorrelation function is also given by the terms with $\mathbf{p}=\mathbf{p}'$.

$$A_l(x-x') = 1 - (1/N^2)\sum_s \sum_{\mathbf{p}} n_{\mathbf{p},s} \times\{\mathbf{p}(\mathbf{r}-\mathbf{r}')/\hbar-(m^2c^2+p^2)^{1/2}(t-t')(c/\hbar)\}^{2\alpha/\pi} \qquad (129)$$

$$=1 \quad -(2/N^2)[V/(2\pi\hbar)^3 \int_{p<p_F} d^3p|(\mathbf{p}+\mathbf{p}_o)(\mathbf{r}-\mathbf{r}')/\hbar-[m^2c^2+(\mathbf{p}+\mathbf{p}_o)^2]^{1/2}(t-t')(c/\hbar)|^{2\alpha/\pi}$$

$$\approx 1 - (1/N)|\mathbf{p}_o(\mathbf{r}-\mathbf{r}')/\hbar-mc^2\tau/\hbar|^{2\alpha/\pi} \qquad \text{for } p_F<<|p_{o3}-mc^2\tau/z|. \qquad (130)$$

Here we have used the mean value theorem, considering the $2\alpha/\pi$ power as a slowly varying function of $\mathbf{p}$ and neglecting p_o in the coefficient of $\tau \equiv t-t'$, with $z\equiv|\mathbf{r}-\mathbf{r}'|$. Writing the power again as an exponential function of its logarithm, expanding the resulting exponential, and keeping only the first term, we obtain with $(\hbar/mc^2)|\mathbf{p}_o(\mathbf{r}-\mathbf{r}')/\hbar-mc^2\tau/\hbar|\equiv\theta=|\tau-\mathbf{p}_o(\mathbf{r}-\mathbf{r}')/mc^2|$

$$A_l(x-x') = 1-[(mc^2/\hbar)\theta]^{2\alpha/\pi}/N$$

$$= (1-2/N) + [\hbar/mc^2]^{2\alpha/\pi}(2\alpha/N\pi)\int_0^\infty \cos[\omega\theta]d\omega/\omega^{1-2\alpha/\pi}.$$

$$\approx \{N-2 + (2\alpha/\pi)\int_0^\infty [\hbar\omega/mc^2]^{2\alpha/\pi}\cos[\omega\theta]\, d\omega/\omega\}/N. \qquad (131)$$

According to the Wiener-Khintchine theorem, the coefficient of the cos gives the spectral density. To get it for the fractional fluctuations $\delta n/n$, we divide by the constant term N-2

$$S_{\delta n/n}(k) \approx [2\alpha/\pi\omega(N-2)][\hbar\omega/mc^2]^{2\alpha/\pi}. \qquad (132)$$

This result is also applicable for the particle current fluctuation spectrum. Indeed, for current density fluctuations δj we include a $(\hbar/mi)\nabla$ in front of each of the two ψ operators in Eq. (127), a factor pp'/p_o^2 in Eq. (128) after the summation signs, a factor $(p/p_o)^2$ in the first form of Eq. (129), a factor $(\mathbf{p}+\mathbf{p}_o)^2/p_o^2$ in the second form, and no changes in Eqs. (130)-(131). Eq. (132) becomes

$$S_{\delta j/j}(k) \approx [2\alpha/\pi\omega(N-2)][\hbar\omega/mc^2]^{2\alpha/\pi}. \qquad (133)$$

This result coincides with our earlier theoretical result for coherent quantum 1/f noise if we replace N with N-2. The validity of this equation is restricted to low frequencies and wave-numbers. This equation is in excellent agreement with mobility and diffusion 1/f noise in large devices.

Finally, we consider the errors caused by the neglect of higher order terms in the expansion of the exponential functions resulting from Eqs. (122)-(123) and (130). For a thermal electron and r=1cm in Eq. (6') we get $kr=10^{12}$ and $(2\alpha/\pi)\ln(kr)=0.12$, yielding an error of 12%. For $t=10^{17}$s in Eq. (130), which is the age of the universe, we get an error of 40% as the upper limit. Therefore a more exact treatment is of some interest. Using the identity [60]

$$\theta^{2\alpha/\pi} \equiv [-(2\alpha/\pi)\int_{\omega_o}^{\infty}\omega^{-2\alpha/\pi}\cos(\theta\omega)d\omega/\omega]$$
$$\times\{\cos\alpha + (2\alpha/\pi)\sum_{n=0}^{\infty}(\theta\omega_o)^{2n-2\alpha/\pi}[(2n)!(2n-2\alpha/\pi)]^{-1}\}^{-1}, \qquad (134)$$

with arbitrarily small cutoff ω_o, we obtain from Eq. (130) the exact form

$$A_j(x-x') = 1 + [(2\alpha/\pi N)\int_{\omega_o}^{\infty}(mc^2/\hbar\omega)^{2\alpha/\pi}\cos(\theta\omega)d\omega/\omega]$$
$$\times\{\cos\alpha + (2\alpha/\pi)\sum_{n=0}^{\infty}(\theta\omega_o)^{2n-2\alpha/\pi}[(2n)!(2n-2\alpha/\pi)]^{-1}\}^{-1}. \qquad (135)$$

This would indicate a $\omega^{-1-2\alpha/\pi}$ spectrum and a 1/N dependence of the spectrum of fractional n and j fluctuations, if we neglect the curly bracket in the denominator which is close to unity for very small ω_o. We thus realize that the unusual N-2 dependence in Eqs. (132)-(133) is caused by the forced introduction of the integrable $\omega^{2\alpha/\pi-1}$ spectrum in place of the $\omega^{-1-2\alpha/\pi}$ spectrum. Due to the smallness of α both forms coincide in practical applications. Eq. (133) for the coherent QED chaos process in electric currents can thus be written also in the form

$$\underline{\underline{S_{\delta j\,/\,j}(k) \approx [2\alpha/\pi\omega N][mc^2/\hbar\omega]^{2\alpha/\pi} \approx 2\alpha/\pi\omega N = 0.00465/\omega N.}} \qquad (136)$$

This result derived directly earlier [61], [62], is in excellent agreement with the measurements [46], in large {see [62] for a definition of "large" or "extended", and for an interpolation with the conventional quantum 1/f chaos effect} devices such as large n^+p Hg_{1-}

$_xCd_xTe$ infrared detector diodes. It is also close to the empirical value of $0.002/\omega N$ observed earlier by Hooge [63] in semiconductors and metals. Being observed in the presence of a constant applied field, these fundamental quantum current fluctuations are usually interpreted as mobility fluctuations.

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"Quantum 1/f Noise in Quartz Resonators" at NIST (Natl. Inst. of Standards), Boulder, Colorado, January 1992.

"Quantum 1/f Effect with High Technology Applications", Dept. of Physics, Univ, of Konstanz, Konstanz, Germany, March 12, 1993.

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"Quantum 1/f Noise with Applications to Infrared Detectors", Electr. Engrg. Dept., Southern Methodist University, Dallas, Oct. 1, 1993.

Service

Organizer and co-Chair of the 4th Symposium on Quantum 1/f Noise and Other Low-Frequency Fluctuations", in Minneapolis, MN on May 22-25, 1992.

Chaired the session on "1/f Noise and Quantum Chaos" at the APS (1992) "March Meeting" in Indianapolis.

Chaired a session at the "XI Int. Conf. on Noise in Physical Systems and 1/f Noise" in Kyoto (Japan) Nov. 1991.

Organizer and Chair of the 5th van der Ziel Symposium on Quantum 1/f Noise and Other Low-Frequency Fluctuations", and of a "Quantum 1/f Retreat" at Univ. of MO St. Louis on May 22-25, 1992.

Chair and Organizer of the "XII Int. Conf. on Noise in Physical Systems and 1/f Noise" in St. Louis, MO, Aug 16-20, 1993.

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